

Section 1.3 Objectives

- Develop algebraic vocabulary and recognize variables, constants, terms, and coefficients.
- Evaluate algebraic expressions given the value of variables.
- Simplify algebraic expressions using the commutative and associative properties.
- Simplify algebraic expressions by combining like terms.
- Simplify algebraic expressions by using the distributive property.

SECTION 1.3 Algebraic Expressions

INTRODUCTION

In this section we make the transition from studying arithmetic to studying algebra. *Algebra* is the branch of mathematics that uses letters (like x or y) to represent unknown values.

You will continue to perform the basic arithmetic operations (addition, subtraction, multiplication, and division) that you reviewed in the last couple sections. And you will continue to work with integers and rational numbers. But included with the operations and numbers may be letters like x or y . Together, they form algebraic expressions. You will learn how to evaluate and simplify algebraic expressions. To begin your study of algebra, it is important to learn some basic vocabulary.

TERMINOLOGY

The first step to learning algebra is to become familiar with the language of algebra. The chart below gives the definitions of important vocabulary words that you should learn.

Vocabulary	Definition	Example
Variable	a letter that represents an unknown quantity	$3\boxed{x} + 5$
Coefficient	a number multiplied with a variable <u>Note:</u> The coefficient is placed to the left of the variable.	$\boxed{3}x + 5$ <u>Note:</u> If no number is in front of the variable, the coefficient is 1. (x is the same as $1x$)
Constant	a number without a variable	$3x + \boxed{5}$
Algebraic Expression	an expression that may contain numbers, variables, and/or math operation symbols <u>Note:</u> An algebraic expression does NOT contain an equal sign.	$3x^2 - 5x + 1$
Term	each part of an algebraic expression separated by addition <u>Note:</u> The terms may also be separated by subtraction since subtracting is the same as adding the opposite.	$3x^2 - 5x + 1$ $= 3x^2 + (-5x) + 1$ There are three terms: $3x^2, -5x, 1$
Variable Term	a term that contains a variable	$3x^2 - 5x + 1$ Variable terms: $3x^2$ and $-5x$

EXAMPLE: For the algebraic expression $3x^2 + 2x - xy + y - 6$, answer the questions below.

$$\begin{aligned} & 3x^2 + 2x - xy + y - 6 \\ & = 3x^2 + 2x + (-xy) + y + (-6) \end{aligned}$$

First rewrite the expression as a sum of terms.

Change the subtractions to adding the opposite.

- | | |
|---|--|
| 1. List the <i>variable(s)</i> . | Answer: x and y |
| 2. List the <i>constant(s)</i> . | Answer: -6 |
| 3. List the <i>term(s)</i> . | Answer: $3x^2$, $2x$, $-xy$, y , and -6 |
| 4. List the <i>variable term(s)</i> . | Answer: $3x^2$, $2x$, $-xy$, y |
| 5. What is the <i>coefficient</i> of the term $2x$? | Answer: 2 |
| 6. What is the <i>coefficient</i> of the term $-xy$? | Answer: -1 |
| 7. What is the <i>coefficient</i> of the term y ? | Answer: 1 |

PRACTICE: For the algebraic expression $2x^3 - 4y + yz + 5$, answer the questions below.

- Write the expression as a sum of terms.
- List the *variable(s)*.
- List the *term(s)*.
- List the *constant(s)*.
- List the *variable term(s)*.
- What is the *coefficient* of the term yz ?
- What is the *coefficient* of the term $2x^3$?
- What is the *coefficient* of the term $-4y$?

Answers:

- | | |
|----------------------------|---------|
| 1. $2x^3 + (-4y) + yz + 5$ | 7. 2 |
| 2. x, y, z | 8. -4 |
| 3. $2x^3, -4y, yz, 5$ | |
| 4. 5 | |
| 5. $2x^3, -4y, yz$ | |
| 6. 1 | |



EVALUATING ALGEBRAIC EXPRESSIONS

Consider the expression $2x + 5$. What does the expression mean? We know that the variable x holds the place for an unknown number. And since there is no operation symbol between the number 2 and the variable x , we know that multiplication is implied. So, the expression $2x + 5$ means “multiply 2 by an unknown number named x and then add 5 to the result.”

Now, suppose we are told that the value of the variable x is the number 3. In this case, what is the value of the algebraic expression $2x + 5$? The process used to determine the answer is called *evaluating the expression*. The process involves replacing the variable x with the number 3 in the algebraic expression and then performing the arithmetic. We show this process below. Notice that the arithmetic is performed using the *Order of Operations* that you learned earlier in this chapter.

$$\begin{aligned}
 2x + 5 & \quad \text{Replace the variable } x \text{ with the number 3.} \\
 = 2(\mathbf{3}) + 5 & \quad \text{Multiply 2 by 3.} \\
 = 6 + 5 & \quad \text{Add 5 to the result.} \\
 = 11 & \quad \text{This is the value of } 2x + 5.
 \end{aligned}$$

So, if $x = 3$, then $2x + 5 = 11$.

EVALUATING ALGEBRAIC EXPRESSIONS

To evaluate an algebraic expression given the value of the variable in the expression,

1. Substitute the given value in place of the variable in the expression.
2. Perform the arithmetic to simplify the expression.
Remember to follow the proper order of operations (PEMDAS).

EXAMPLES: Evaluate each expression.

1. Evaluate $(x+3)+10$ if $x = -5$.

$$\begin{aligned}
 (x+3) + 10 & \quad \text{Substitute } -5 \text{ for } x. \\
 = (-\mathbf{5}+3) + 10 & \quad \text{Add the integers inside the parentheses.} \\
 = (-2) + 10 & \quad \text{Add.} \\
 = 8 & \quad \text{This is the answer.}
 \end{aligned}$$

2. Evaluate $x(2+x)$ if $x = 4$.

$$\begin{aligned}
 x(2+x) & \quad \text{Substitute 4 for each } x. \\
 = \mathbf{4}(2+4) & \quad \text{Add the integers inside the parentheses.} \\
 = 4(\mathbf{6}) & \quad \text{Multiply.} \\
 = 24 & \quad \text{This is the answer.}
 \end{aligned}$$

3. Evaluate $\frac{1}{2}(x+4)$ if $x=8$.

$$\begin{aligned} & \frac{1}{2}(x+4) && \text{Substitute 8 for } x. \\ = & \frac{1}{2}(8+4) && \text{Add the integers inside the parentheses.} \\ = & \frac{1}{2}(12) && \text{Write 12 as a fraction in order to multiply.} \\ = & \frac{1}{2} \cdot \frac{12}{1} && \text{Divide out common factors in the numerator and denominator.} \\ = & \frac{1}{\cancel{2}} \cdot \frac{\cancel{12}^6}{1} && \text{Multiply.} \\ = & \frac{6}{1} && \text{Simplify.} \\ = & 6 && \text{This is the answer.} \end{aligned}$$

4. Evaluate $5x+2y$ if $x=-2$ and $y=-3$.

$$\begin{aligned} & 5x + 2y && \text{Substitute } -2 \text{ for } x \text{ and } -3 \text{ for } y. \\ = & 5(-2) + 2(-3) && \text{Multiply.} \\ = & -10 + (-6) && \text{Add.} \\ = & -16 && \text{This is the answer.} \end{aligned}$$

5. Evaluate $3x+8y$ if $x=\frac{2}{3}$ and $y=-1$.

$$\begin{aligned} & 3x + 8y && \text{Substitute } \frac{2}{3} \text{ for } x \text{ and } -1 \text{ for } y. \\ = & 3\left(\frac{2}{3}\right) + 8(-1) && \text{Write 3 as the fraction } \frac{3}{1} \text{ in order to multiply by } \frac{2}{3}. \\ & && \text{Multiply 8 by } -1. \\ = & \frac{3}{1} \cdot \frac{2}{3} + (-8) && \text{Divide out common factors in the numerator and denominator of the fractions.} \\ = & \frac{\cancel{3}^1}{1} \cdot \frac{2}{\cancel{3}_1} + (-8) && \text{Multiply the fractions.} \\ = & 2 + (-8) && \text{Add.} \\ = & -6 && \text{This is the answer.} \end{aligned}$$

6. Evaluate $(x+y)^2$ if $x=-6$ and $y=4$.

$$\begin{aligned} & (x+y)^2 && \text{Substitute } -6 \text{ for } x \text{ and } 4 \text{ for } y. \\ = & (-6+4)^2 && \text{Add the integers inside the parentheses.} \\ = & (-2)^2 && \text{Multiply } (-2) \text{ two times.} \\ = & (-2)(-2) && \text{Multiply.} \\ = & 4 && \text{This is the answer.} \end{aligned}$$

7. Evaluate x^2 if $x = -3$.

$$\begin{aligned}
 & x^2 && \text{Rewrite with the variable in parentheses (exponent outside).} \\
 = & (x)^2 && \text{Substitute } -3 \text{ for } x. \\
 = & (-3)^2 && \text{Multiply } (-3) \text{ two times.} \\
 = & (-3)(-3) && \text{Multiply.} \\
 = & 9 && \text{This is the answer.}
 \end{aligned}$$

8. Evaluate $-x^2 - 3x$ if $x = -2$.

$$\begin{aligned}
 & -x^2 - 3x && \text{Rewrite with parentheses around the variables.} \\
 = & -(x)^2 - 3(x) && \text{Substitute } -2 \text{ for each } x. \\
 = & -(-2)^2 - 3(-2) && \text{Write } (-2)^2 \text{ in expanded form.} \\
 = & -(-2)(-2) - 3(-2) && \text{Multiply } (-2) \text{ by } (-2). \text{ Multiply } 3 \text{ by } (-2). \\
 = & -(4) - (-6) && \text{Change subtraction to adding the opposite.} \\
 = & -4 + 6 && \text{Add.} \\
 = & 2 && \text{This is the answer.}
 \end{aligned}$$

9. Evaluate $\frac{45}{a} - \frac{6}{b} + c$ if $a = -9$, $b = 12$, and $c = -2$.

$$\begin{aligned}
 & \frac{45}{a} - \frac{6}{b} + c && \text{Substitute } -9 \text{ for } a, 12 \text{ for } b, \text{ and } -2 \text{ for } c. \\
 = & \frac{45}{-9} - \frac{6}{12} + (-2) && \begin{array}{l} \text{Simplify the first fraction to get } -5. \\ \text{Simplify the second fraction to get } \frac{1}{2}. \end{array} \\
 = & -5 - \frac{1}{2} + (-2) && \text{Rearrange the second and third terms so that the integers are next to each other.} \\
 = & -5 + (-2) - \frac{1}{2} && \text{Add the integers.} \\
 = & -7 - \frac{1}{2} && \text{Change the subtraction to adding the opposite.} \\
 = & -7 + \left(-\frac{1}{2}\right) && \text{Add the two remaining terms.} \\
 = & -7\frac{1}{2} && \text{Express the mixed number as an improper fraction.} \\
 = & -\frac{15}{2} && \text{This is the answer.}
 \end{aligned}$$

10. Evaluate $\frac{5x+y-3}{4x-2y}$ if $x=2$ and $y=-1$.

$$\begin{aligned} & \frac{5x+y-3}{4x-2y} \\ &= \frac{5(2)+(-1)-3}{4(2)-2(-1)} \\ &= \frac{10+(-1)-3}{4(2)-2(-1)} \\ &= \frac{9-3}{4(2)-2(-1)} \\ &= \frac{6}{4(2)-2(-1)} \\ &= \frac{6}{8-(-2)} \\ &= \frac{6}{8+2} \\ &= \frac{6}{10} \\ &= \frac{3}{5} \end{aligned}$$

Substitute 2 for each x and -1 for each y .

Numerator: Simplify by following the order of operations:

Multiply 5 and 2 to get 10.

Add 10 and -1 to get 9.

Subtract to get 6.

Denominator: Simplify by following the order of operations.

Multiply 4 and 2 to get 8. Multiply 2 and -1 to get -2 .

Change the subtraction to adding the opposite.

Add to get 10.

Simplify the fraction by dividing the numerator and denominator by 2.

This is the answer.

PRACTICE: Evaluate each expression.

1. Evaluate $4x-7$ if $x=3$.

2. Evaluate $\frac{1}{3}(x+4)$ if $x=2$.

3. Evaluate $(a-3)+21$ if $a=-6$.

4. Evaluate $y(y-7)$ if $y=3$.

5. Evaluate $4x+2y$ if $x=6$ and $y=5$.

6. Evaluate $x+6y$ if $x=-4$ and $y=-7$.

7. Evaluate $4x+3y$ if $x=\frac{1}{2}$ and $y=-1$.

8. Evaluate $8x+9y$ if $x=\frac{1}{4}$ and $y=-2$.

9. Evaluate $\frac{2}{3}(2x+6y)$ if $x=3$ and $y=1$.

10. Evaluate x^2 if $x=-5$.






11. Evaluate $(2x-y)^2$ if $x=-2$ and $y=5$.

12. Evaluate x^2-5x if $x=-3$.

13. Evaluate $\frac{36}{a}-\frac{14}{b}+c$ if $a=-9$, $b=7$, and $c=-3$.

14. Evaluate $\frac{3x-y+2}{4x+5y}$ if $x=2$ and $y=5$.

Answers:

- | | | | | | |
|----|-----|---|-----|----------------|---|
| 1. | 5 |  | 8. | -16 |  |
| 2. | 2 |  | 9. | 8 |  |
| 3. | 12 |  | 10. | 25 |  |
| 4. | -12 |  | 11. | 81 |  |
| 5. | 34 |  | 12. | 24 |  |
| 6. | -46 |  | 13. | -9 |  |
| 7. | -1 |  | 14. | $\frac{1}{11}$ |  |

ALGEBRAIC PROPERTIES

There are a couple properties that we will use as we continue our study of algebra. These properties are important because they allow us to change the order or grouping of terms in algebraic expressions.

COMMUTATIVE PROPERTY OF ADDITION		
WORDS	SYMBOLS	EXAMPLE
Changing the order in which two terms are added does not change the sum (answer).	$a + b = b + a$	$2 + 6 = 6 + 2$ $8 = 8$

COMMUTATIVE PROPERTY OF MULTIPLICATION		
WORDS	SYMBOLS	EXAMPLE
Changing the order in which two factors are multiplied does not change the product (answer).	$a \cdot b = b \cdot a$	$2 \cdot 6 = 6 \cdot 2$ $12 = 12$

NOTE: The operations of subtraction and division do NOT have a *Commutative Property*.

Subtraction is NOT commutative.

$$8 - 5 \neq 5 - 8$$

$$3 \neq -3$$

Division is NOT commutative.

$$10 \div 5 \neq 5 \div 10$$

$$2 \neq 0.5$$

EXAMPLES: Rewrite each expression using the *Commutative Property of Addition or Multiplication*.

1. $x + 5$ Change the **order** of the terms.
 $= 5 + x$ This is the answer.

2. yx Change the **order** of the factors.
 $= xy$ This is the answer.

3. $4 + 9x$ Change the **order** of the terms.
 $= 9x + 4$ This is the answer.

4. $(x)(8)$ Change the **order** of the factors.
 $= (8)(x)$ Remove the parentheses; 8 is the coefficient of x , so multiplication is implied.
 $= 8x$ This is the answer.

5. $2 - 3x$ Since there is no commutative property for subtraction, we must first rewrite the expression by changing the subtraction to adding the opposite.
 $= 2 + (-3x)$ Rewrite the addition expression using the commutative property.
 Change the **order** of the terms.
 $= -3x + 2$ This is the answer.

REVIEW: COMMUTATIVE PROPERTY

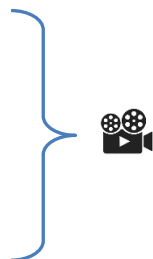


PRACTICE: Rewrite each expression using the *Commutative Property of Addition or Multiplication*.

1. $y + 9$
2. $(c)(-7)$
3. $-3 + 6w$
4. $(a)(c)$
5. $4x - 5$

Answers:

1. $9 + y$
2. $-7c$
3. $6w - 3$
4. $(c)(a)$ OR ca
5. $-5 + 4x$



ASSOCIATIVE PROPERTY OF ADDITION		
WORDS	SYMBOLS	EXAMPLE
Changing the grouping of terms being added does not change the sum (answer).	$(a+b)+c = a+(b+c)$ Notice that the order of the terms is the same on both sides of the equation.	$(4+6)+2 = 4+(6+2)$ $10+2 = 4+8$ $12 = 12$

ASSOCIATIVE PROPERTY OF MULTIPLICATION		
WORDS	SYMBOLS	EXAMPLE
Changing the grouping of factors being multiplied does not change the product (answer).	$(ab)c = a(bc)$ Notice that the order of the factors is the same on both sides of the equation.	$(4 \cdot 6) \cdot 2 = 4 \cdot (6 \cdot 2)$ $24 \cdot 2 = 4 \cdot 12$ $48 = 48$

NOTE: The operations of subtraction and division do NOT have an *Associative Property*.

Subtraction is NOT associative.

$$100 - (30 - 10) \neq (100 - 30) - 10$$

$$100 - 20 \neq 70 - 10$$

$$80 \neq 60$$

Division is NOT associative.

$$64 \div (8 \div 4) \neq (64 \div 8) \div 4$$

$$64 \div 2 \neq 8 \div 4$$

$$32 \neq 2$$

EXAMPLES: Rewrite each expression using the *Associative Property of Addition or Multiplication*.

- $(7+x)+8$ Leave the order of the terms the same. Change the **grouping** (the parentheses).
 $= 7+(x+8)$ This is the answer.
- $3 \cdot (9 \cdot y)$ Leave the order of the factors the same. Change the **grouping** (the parentheses).
 $= (3 \cdot 9) \cdot y$ This is the answer.

REVIEW: ASSOCIATIVE PROPERTY 

PRACTICE: Rewrite each expression using the *Associative Property of Addition or Multiplication*.

- $(y+8)+4$
- $5 \cdot (7 \cdot a)$

Answers:

- $y+(8+4)$ 
- $(5 \cdot 7) \cdot a$ 

SIMPLIFYING ALGEBRAIC EXPRESSIONS

Earlier in this section you learned to *evaluate* algebraic expressions. Now you will learn to *simplify* algebraic expressions. Simplifying algebraic expressions involves using the properties you just studied, combining like (similar) terms, and applying a property called the Distributive Property.

SIMPLIFYING ALGEBRAIC EXPRESSIONS USING THE COMMUTATIVE AND ASSOCIATIVE PROPERTIES

The properties you just learned can be used to reorder or regroup terms in an algebraic expression. This makes it possible to combine terms and rewrite the expression in its simplest form.

EXAMPLES: Use the Commutative or Associative Property to rewrite and simplify each expression.

- $3 + 5x + 6$ Use the Commutative Property.
 $= 3 + 5x + 6$ Change the order of the first and second terms so the constants will be next to each other.
 $= 5x + 3 + 6$ Leave the $5x$. Add $3 + 6$.
 $= 5x + 9$ This is the simplified expression.
- $7 \cdot \left(\frac{1}{7} \cdot x\right)$ Use the Associative Property.
 $= 7 \cdot \left(\frac{1}{7} \cdot x\right)$ Change the grouping (parentheses) so that the numbers will be grouped together.
 $= \left(7 \cdot \frac{1}{7}\right) \cdot x$ Write the integer as a fraction.
 $= \left(\frac{7}{1} \cdot \frac{1}{7}\right) \cdot x$ Divide out common factors. Then multiply.
 $= 1 \cdot x$ Multiply 1 and x to get x .
 $= x$ This is the simplified expression.

PRACTICE: Use the Commutative or Associative Property to rewrite and simplify each expression.

- $2 \cdot x \cdot 6$
- $\frac{2}{3} + 8x + \frac{1}{3}$
- $(4x + 8) + 1$
- $\frac{5}{6} \cdot \left(\frac{6}{5} \cdot x\right)$

Answers:

- $12x$
- $8x + 1$
- $4x + 9$
- x

SIMPLIFYING ALGEBRAIC EXPRESSIONS BY COMBINING LIKE TERMS

On the previous page, using the properties allowed you to combine *constants* (numbers). In algebra, other terms can be combined as well. Algebraic expressions can be simplified by adding and subtracting *like terms*. Specifically, *like terms* are terms that have the same variable(s) raised to the same exponent(s). For example, $6x^3$ and $2x^3$ are like terms because they both contain the variable x , and they both contain the exponent 3. The coefficients of the terms can be different. In this example, the coefficients are 6 and 2.

LIKE TERMS

Terms that have the same variable(s) raised to the same exponent(s).

Note: The coefficients can be different.

In order to simplify expressions by combining like terms, you must first be able to identify like terms. We focus on that in the examples below.

EXAMPLES: Determine if the following pairs of terms are like terms.

- | | | | | |
|----|------------------------|---|---------------------|--|
| 1. | $2a$ and $3a$ | → | <i>Like terms</i> | The variables are the same and the exponents are the same. |
| 2. | $2x$ and $3y$ | → | <i>Unlike terms</i> | The variables are not the same. |
| 3. | $2x$ and 3 | → | <i>Unlike terms</i> | One has the variable x . The other does not. |
| 4. | x^2 and x | → | <i>Unlike terms</i> | The exponents are not the same. |
| 5. | x^2 and $0.5x^2$ | → | <i>Like terms</i> | The variables are the same and the exponents are the same. |
| 6. | x and $\frac{1}{3}x$ | → | <i>Like terms</i> | The variables are the same and the exponents are the same. |
| 7. | 4 and 2 | → | <i>Like terms</i> | Constants are also like terms. |
| 8. | $3ab$ and $2b$ | → | <i>Unlike terms</i> | The variables are not the same. |

REVIEW: LIKE TERMS



PRACTICE: Determine if the following pairs of terms are like terms.

- | | |
|--------------------|------------------------------|
| 1. $4x$ and $6x$ | 4. $\frac{1}{2}y$ and $1.2y$ |
| 2. a^2 and a^3 | 5. $-x^2y$ and $3xy$ |
| 3. $3b$ and 3 | 6. 3 and 4 |

Answers:



- | | |
|-----------------|-----------------|
| 1. Like Terms | 4. Like Terms |
| 2. Unlike Terms | 5. Unlike Terms |
| 3. Unlike Terms | 6. Like Terms |

Once like terms have been identified in an algebraic expression, you can simplify the expression by combining the like terms. When you combine like terms, you add or subtract the coefficients of the terms. However, the variable stays the same and the exponent stays the same.

Some steps of the process involve reordering or regrouping terms. Although it may not be specifically stated, keep in mind that, whenever we reorder or regroup, we are using the Commutative and Associative Properties.

SIMPLIFYING EXPRESSIONS BY COMBINING LIKE TERMS

1. Reorder the terms so that *like terms* are next to each other.
2. Add or subtract the *coefficients* of the like terms.
3. Keep the *variable* and the *exponent* the same.

EXAMPLES: Simplify each expression.

- | | |
|--|--|
| 1. $2x + 3x$
$= (2 + 3)x$
$= 5x$ | These are like terms.
Add the coefficients. Keep the variable and exponent the same.
This is the simplified expression. |
| 2. $2y + 5$ | These two terms are <u>unlike</u> , so they cannot be combined. |
| 3. $3x + 4y - 10x + 7y$
$= 3x - 10x + 4y + 7y$
$= \underbrace{(3 - 10)x} + \underbrace{(4 + 7)y}$
$= -7x + 11y$ | Identify the like terms.
Reorder the terms so that like terms are next to each other.
Subtract the coefficients of the “x terms.” (<i>Or change the subtraction to adding the opposite: $3 + -10$</i>). Keep the variable and the exponent the same.
Add the coefficients of the “y terms.” Keep the variable and exponent the same.
This is the simplified expression. |
| 4. $-2x + 4 - 7 + 8x + 11x - 6$
$= -2x + 8x + 11x + 4 - 7 - 6$
$= \underbrace{(-2 + 8 + 11)x} + \underbrace{(4 - 7 - 6)}$
$= 17x + (-9)$
$= 17x - 9$ | Identify the like terms.
Reorder the terms so that like terms are next to each other.
Add the coefficients of the “x terms.” Keep the variable and exponent the same.
Subtract the constants. (<i>Or change the subtractions to adding the opposite.</i>)
Rewrite as a subtraction problem.
This is the simplified expression. |

5.	$3xy - y - 4xy + 5y$	Identify the like terms.
	$= 3xy - 4xy - y + 5y$	Reorder the terms so that like terms are next to each other.
	$= 3xy - 4xy - 1y + 5y$	The third term is $-y$ which means $-1y$.
	$= \underbrace{(3-4)xy} + \underbrace{(-1+5)y}$	Subtract the coefficients of the “ xy terms.” (Or change the subtraction to adding the opposite.)
	$= -1xy + 4y$	Add the coefficients of the “ y terms.”
	$= -xy + 4y$	Rewrite the first term without the 1. Both are correct, but $-xy$ is preferred.
		This is the simplified expression.

As you become more comfortable with the process of combining like terms, you may choose to skip the step with the parentheses.

For example, instead of writing $2x + 7x + 10y - 6y$, you can simply write $\underbrace{2x + 7x}_{9x} + \underbrace{10y - 6y}_{4y}$.

This shorter way is shown below as we simplify algebraic expressions that contain fractions.

EXAMPLES: Simplify each expression.

1.	$\frac{2}{5}x + \frac{2}{15}x$	These are like terms. To combine them, we need to add the coefficients. But since the coefficients are fractions, we need a common denominator.
	$= \frac{2 \cdot 3}{5 \cdot 3}x + \frac{2}{15}x$	To get a common denominator of 15, we multiply the numerator and denominator of the first fraction by 3.
	$= \frac{6}{15}x + \frac{2}{15}x$	Add the coefficients.
	$= \frac{8}{15}x$	This is the simplified expression.
2.	$\frac{1}{4}x - \frac{8}{12} + \frac{3}{5}x + \frac{5}{6}$	Identify like terms.
	$= \frac{1}{4}x + \frac{3}{5}x - \frac{8}{12} + \frac{5}{6}$	Reorder the terms so that like terms are next to each other.
	$= \frac{1 \cdot 5}{4 \cdot 5}x + \frac{3 \cdot 4}{5 \cdot 4}x - \frac{8}{12} + \frac{5 \cdot 2}{6 \cdot 2}$	Get a common denominator of 20 for the “ x terms.” Get a common denominator of 12 for the constants.
	$= \underbrace{\frac{5}{20}x + \frac{12}{20}x} - \frac{8}{12} + \frac{10}{12}$	Add the coefficients of the “ x terms.” Add the constants.
	$= \frac{17}{20}x + \frac{2}{12}$	Simplify the second fraction.
	$= \frac{17}{20}x + \frac{1}{6}$	This is the simplified expression.

PRACTICE: Simplify each expression.

1. $8y - 3y$

2. $9a + 6$

3. $3x - 4x$

4. $2x + 7x - 5 + 4$

5. $5a - 83 - 13a$

6. $4x - 5 - 6x - 3$

7. $8x - 3y + 2x - 5y$

8. $-6xy + 5y - 2xy - 7y$

9. $-7a + 3 + 1 + a + 4a - 5$

10. $\frac{1}{8}x - \frac{4}{3}x$

11. $\frac{2}{3}x - \frac{1}{6}x + 2 + \frac{4}{5}$

12. $\frac{1}{5}x - \frac{1}{3} + \frac{2}{15}x - \frac{1}{2}$

Answers:

1. $5y$



2. $9a + 6$



3. $-x$



4. $9x - 1$



5. $-8a - 83$



6. $-2x - 8$



7. $10x - 8y$



8. $-8xy - 2y$

9. $-2a - 1$



10. $-\frac{29}{24}x$



11. $\frac{1}{2}x + \frac{14}{5}$




12. $\frac{1}{3}x - \frac{5}{6}$


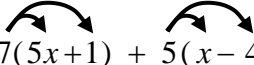
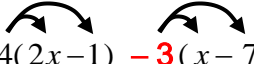
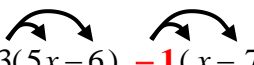
SIMPLIFYING ALGEBRAIC EXPRESSIONS USING THE DISTRIBUTIVE PROPERTY

The *Distributive Property* is used to simplify algebraic expressions with parentheses. When you studied the Order of Operations, you learned that operations inside parentheses must be completed first. But if you have an expression like $2(3x + 8)$, you cannot add the terms inside the parentheses because they are not like terms. With expressions such as this, the *Distributive Property* is used.

Let's explain how the *Distributive Property* works by using the expression $a(b + c)$. The property allows us to multiply the term outside the parentheses by each term inside the parentheses. So, to simplify the expression $a(b + c)$, first distribute the a to the b . This means to multiply a by b , which gives ab . Then distribute the a to the c . This means to multiply a by c , which gives ac . The result is that the parentheses are eliminated from the expression, giving us $ab + ac$. After you apply the *Distributive Property*, it is important to combine any like terms if possible.

DISTRIBUTIVE PROPERTY	
Multiply the term outside the parentheses by each term inside the parentheses.	For any real numbers a , b , and c ,  $a(b + c) = ab + ac$

EXAMPLES: Simplify each expression.

1. $2(3x+8)$

 $= 2(3x+8)$
 $= 2 \cdot 3x + 2 \cdot 8$
 $= 6x + 16$
- We will use the Distributive Property to clear the parentheses.
 Distribute the 2 outside the parentheses to each term inside the parentheses.
 Multiply 2 by 3x and multiply 2 by 8.
 This is the simplified expression.
2. $7(5x+1) + 5(x-4)$

 $= 7(5x+1) + 5(x-4)$
 $= 35x+7 + 5x-20$
 $= \underbrace{35x + 5x} + \underbrace{7 - 20}$
 $= 40x - 13$
- We will use the Distributive Property to clear the parentheses.
 Distribute the 7 by multiplication to the 5x and to the 1.
 Distribute the 5 by multiplication to the x and to the -4.
 Rewrite the expression with the like terms next to each other.
 Combine like terms.
 This is the simplified expression.
3. $4(2x-1) - 3(x-7)$

 $= 4(2x-1) - 3(x-7)$
 $= 8x - 4 - 3x + 21$
 $= \underbrace{8x - 3x} - \underbrace{4 + 21}$
 $= 5x + 17$
- We will use the Distributive Property to clear the parentheses.
 Distribute the 4 by multiplication to the 2x and to the -1.
 Notice that the second set of parentheses is preceded by a **negative** number.
 Distribute the -3 by multiplication to the x and to the -7.
 Rewrite the expression with the like terms next to each other.
 Combine like terms.
 This is the simplified expression.
4. $3(5x-6) - (x-7)$

 $= 3(5x-6) - 1(x-7)$
 $= 15x - 18 - 1x + 7$
 $= \underbrace{15x - 1x} - \underbrace{18 + 7}$
 $= 14x - 11$
- We will use the Distributive Property to clear the parentheses.
 Distribute the 3 by multiplication to the 5x and to the -6.
 Since the second set of parentheses is preceded by a **minus** sign, the number that will be distributed through the second set of parentheses is -1.
 Distribute the -1 by multiplication to the x and to the -7.
 Rewrite the expression with the like terms next to each other.
 Combine like terms.
 This is the simplified expression.

$$5. \quad 5 - 6(x + 3)$$

$$= 5 - 6(x + 3)$$

$$= 5 - 6x - 18$$

$$= -6x + \underbrace{5 - 18}$$

$$= -6x - 13$$

DO NOT BEGIN BY SUBTRACTING $5 - 6$!

Remember the Order of Operations, PEMDAS. It states that we must perform multiplication before subtraction.

So, the first step is to multiply the -6 outside the parentheses by each term inside the parentheses.

Distribute the -6 by multiplication to the x and to the 3 .

Rewrite the expression with the like terms next to each other.

Combine like terms.

This is the simplified expression.

PRACTICE: Simplify each expression.

$$1. \quad 5(x - 7)$$

$$2. \quad 4(y + 1)$$

$$3. \quad -2(9a + 4)$$

$$4. \quad 3.9(x - 1.5)$$

$$5. \quad -(3c - 7)$$

$$6. \quad -(2x + 3) - 4$$

$$7. \quad 4 + 2(3x - 1)$$

$$8. \quad 6(x - 8) + 2(3x + 9)$$

$$9. \quad 3(5x + 6) - 7(4x - 2)$$

$$10. \quad -8(2x - 5) - (x + 10)$$

Answers:

$$1. \quad 5x - 35$$

$$2. \quad 4y + 4$$

$$3. \quad -18a - 8$$

$$4. \quad 3.9x - 5.85$$

$$5. \quad -3c + 7$$

$$6. \quad -2x - 7$$

$$7. \quad 6x + 2$$

$$8. \quad 12x - 30$$

$$9. \quad -13x + 32$$

$$10. \quad -17x + 30$$

SIMPLIFYING RATIONAL EXPRESSIONS USING THE DISTRIBUTIVE PROPERTY

Now you will continue to simplify expressions using the Distributive Property. The only difference is that the expressions contain one or more fractions. Don't panic! Simplifying expressions with rational numbers is done the same way as simplifying expressions with integers. You will use the Distributive Property to "clear" the parentheses just as you did before. To make it easier to work with the fractions, it is a good idea to write out the step showing the multiplication of the term outside the parentheses by each of the terms inside.

EXAMPLES: Simplify each expression.

1.
$$\frac{1}{4}(8x - 12) + \frac{2}{5}(15x + 10)$$

Use the Distributive Property to clear the parentheses.

Distribute $\frac{1}{4}$ through the 1st set of parentheses.
Distribute $\frac{2}{5}$ through the 2nd set of parentheses.

$$= \frac{1}{4}(8x - 12) + \frac{2}{5}(15x + 10)$$

Multiply $\frac{1}{4}$ by $8x$ and by -12 .
Multiply $\frac{2}{5}$ by $15x$ and by 10 .

$$= \left(\frac{1}{4} \cdot 8x\right) + \left(\frac{1}{4} \cdot -12\right) + \left(\frac{2}{5} \cdot 15x\right) + \left(\frac{2}{5} \cdot 10\right)$$

Write all terms as fractions.
Divide out common factors.
Multiply and simplify in each set of parentheses.

$$= \left(\frac{\cancel{4}_1 \cdot \overset{2}{\cancel{8}}x}{\cancel{4}_1}\right) + \left(\frac{\cancel{4}_1 \cdot \overset{-3}{\cancel{12}}}{\cancel{4}_1}\right) + \left(\frac{\cancel{2}^3 \cdot \overset{3}{\cancel{15}}x}{\cancel{5}_1}\right) + \left(\frac{\cancel{2}^2 \cdot \overset{2}{\cancel{10}}}{\cancel{5}_1}\right)$$

Rewrite with like terms next to each other.

$$= 2x + (-3) + 6x + 4$$

Combine like terms.

$$= 2x + 6x + (-3) + 4$$

This is the simplified expression.

$$= 8x + 1$$

2.
$$\frac{2}{3}(6x - 9) - \frac{3}{4}(12x - 4)$$

Use the Distributive Property to clear the parentheses.

Distribute $\frac{2}{3}$ through the 1st set of parentheses.
Distribute $-\frac{3}{4}$ through the 2nd set of parentheses.

$$= \frac{2}{3}(6x - 9) - \frac{3}{4}(12x - 4)$$

Multiply $\frac{2}{3}$ by $6x$ and by -9 .
Multiply $-\frac{3}{4}$ by $12x$ and by -4 .

$$= \left(\frac{2}{3} \cdot 6x\right) + \left(\frac{2}{3} \cdot -9\right) + \left(-\frac{3}{4} \cdot 12x\right) + \left(-\frac{3}{4} \cdot -4\right)$$

Write all terms as fractions.
Divide out common factors.
Multiply and simplify in each set of parentheses.

$$= \left(\frac{\cancel{2}^2 \cdot \overset{2}{\cancel{6}}x}{\cancel{3}_1}\right) + \left(\frac{\cancel{2}^3 \cdot \overset{-3}{\cancel{9}}}{\cancel{3}_1}\right) + \left(\frac{\cancel{-3}^3 \cdot \overset{3}{\cancel{12}}x}{\cancel{4}_1}\right) + \left(\frac{\cancel{-3}^1 \cdot \overset{-1}{\cancel{4}}}{\cancel{4}_1}\right)$$

Rewrite with like terms next to each other.

$$= 4x + (-6) + (-9x) + 3$$

Combine like terms.

$$= 4x + (-9x) + (-6) + 3$$

This is the simplified expression.

$$= -5x - 3$$

PRACTICE: Simplify each expression.

1. $\frac{3}{5}(8x+5)$

4. $\frac{1}{3}(3x-9)-\frac{1}{4}(16x+8)$


2. $-\frac{3}{4}\left(6h-\frac{8}{9}k\right)$


5. $\frac{1}{2}(10x+6)+\frac{3}{8}(16x-24)$


3. $\frac{5}{6}\left(\frac{18}{25}x-12\right)$

6. $7(x+3)-\frac{2}{9}(36x-27)$

Answers:

1. $\frac{24}{5}x+3$ 

4. $-3x-5$ 

2. $-\frac{9}{2}h+\frac{2}{3}k$ 

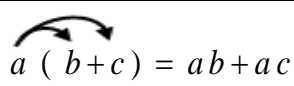

5. $11x-6$

3. $\frac{3}{5}x-10$

6. $-x+27$

SECTION 1.3 SUMMARY

Algebraic Expressions

ALGEBRA VOCABULARY	<p>ALGEBRAIC EXPRESSION</p> <ul style="list-style-type: none"> may contain numbers, variables, and/or math operation symbols does NOT contain an equal sign 	<p><u>Example:</u> $2x + 6$</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> $2x$ Variable Term </div> <div style="text-align: center;"> $+$ </div> <div style="text-align: center;"> 6 Constant Term </div> </div>	<p>$2x + 6$</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> 2 Coefficient </div> <div style="text-align: center;"> x Variable </div> <div style="text-align: center;"> $+$ </div> <div style="text-align: center;"> 6 Constant </div> </div>	
EVALUATING ALGEBRAIC EXPRESSIONS	<ol style="list-style-type: none"> Replace the variable(s) with the given value(s). Perform the arithmetic using the proper order of operations (PEMDAS). 	<p><u>Example:</u> Evaluate $7x - 2y$ if $x = 5$ and $y = 3$.</p> $= 7(5) - 2(3)$ $= 35 - 6$ $= 29$		
COMMUTATIVE & ASSOCIATIVE PROPERTIES	PROPERTY	DESCRIPTION	ADDITION	MULTIPLICATION
	Commutative	Reordering does not change the answer.	$a + b = b + a$ $4 + 2 = 2 + 4$	$a \cdot b = b \cdot a$ $2 \cdot 3 = 3 \cdot 2$
	Associative	Regrouping does not change the answer.	$(a + b) + c = a + (b + c)$ $(5 + 8) + 1 = 5 + (8 + 1)$	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$ $(5 \cdot 2) \cdot 3 = 5 \cdot (2 \cdot 3)$
	<p>SIMPLIFYING EXPRESSIONS USING THE COMMUTATIVE PROPERTY</p> <p>Reorder the terms, then simplify.</p>		<p><u>Example:</u></p> $8 + 3x - 1$ $= 3x + 8 - 1$ $= 3x + 7$	
<p>SIMPLIFYING EXPRESSIONS USING THE ASSOCIATIVE PROPERTY</p> <p>Regroup the terms, then simplify.</p>		<p><u>Example:</u></p> $\frac{1}{3} \cdot \left(\frac{3}{2} \cdot x\right)$ $= \left(\frac{1}{3} \cdot \frac{3}{2}\right) \cdot x$ $= \frac{1}{2}x$		
LIKE TERMS	<p>LIKE TERMS – terms that have the same variable(s) raised to the same exponent(s)</p>	<p><u>Examples:</u></p> <p>Like Terms: $3x$ and $5x$</p> <p>Like Terms: $3x^2$ and $5x^2$</p>	<p>Unlike Terms: $3x$ and $5y$</p> <p>Unlike Terms: $3x$ and $5x^2$</p>	
	<p>SIMPLIFYING EXPRESSIONS BY COMBINING LIKE TERMS</p> <ol style="list-style-type: none"> Reorder so like terms are next to each other. Add or subtract the coefficients of the like terms. Keep the variable and exponent the same. 		<p><u>Example:</u></p> $4x + 3y + 5x - 8y$ $= \underbrace{4x + 5x} + \underbrace{3y - 8y}$ $= 9x - 5y$	
DISTRIBUTIVE PROPERTY	<p>DISTRIBUTIVE PROPERTY</p>	$a(b + c) = ab + ac$ 		
	<p>SIMPLIFYING EXPRESSIONS USING THE DISTRIBUTIVE PROPERTY</p> <p>Multiply the term outside the parentheses by each term inside the parentheses.</p>		<p><u>Example:</u></p> $2(3x + 4)$ $= 2(3x) + 2(4)$ $= 6x + 8$ 	

SECTION 1.3 EXERCISES**Algebraic Expressions**

Evaluate each expression.

1. $2(x+9)$ if $x = -6$
2. $x^3 + 4$ if $x = -4$
3. $2a - 9b$ if $a = -\frac{1}{2}$ and $b = \frac{2}{3}$
4. $\frac{3x - y + 4}{xy - 5}$ if $x = -2$ and $y = 3$
5. $(-3x + y)^2$ if $x = 4$ and $y = 7$
6. $\frac{21}{a} - \frac{8}{b} + c$ if $a = -7$, $b = -4$, and $c = 5$
7. $y^2 + 3y$ if $y = -5$
8. $\frac{4}{5}(x - 3)$ if $x = -2$
9. $7x + 4y$ if $x = -3$ and $y = 4$
10. $-9x^2 + 5y$ if $x = -1$ and $y = -8$
11. $6x - 9y$ if $x = \frac{1}{3}$ and $y = -\frac{5}{9}$
12. $-2.5(x + 7)$ if $x = 2$

Use the Commutative or Associative Property to rewrite and simplify each expression.

13. $8 + 5x + 3$
14. $9 \cdot x \cdot 2$
15. $(6x + 2) + 7$
16. $4 \cdot \left(\frac{1}{4} \cdot a\right)$

Simplify each expression.

17. $7x - 3x - 8 + 2$

18. $8x - 3y - 5x + 7y$

19. $10 + x - 11x - 11 + 2x + 12$

20. $7xy + 3x - 6xy - 5x$

21. $-4a + 3 - 8b + 6a - b$

22. $3.5x - 4y + 2y - 7.5x$

23. $\frac{1}{8}x + \frac{1}{3}x + \frac{3}{5} - \frac{1}{4}$

24. $\frac{2}{5} - \frac{8}{20}x + \frac{3}{15} - \frac{1}{4}x$

Simplify each expression.

25. $6(8a - 4)$

26. $-3.9(x + 1.2)$

27. $-(5x - 7) + 8$

28. $3 + 6(-2x + 6)$

29. $6 - 7(x - 1)$

30. $5(x + 2) + 3(8x + 7)$

31. $8(4x - 2) - 9(3x + 8)$

32. $-4(x + 2) - (7x - 13)$

33. $2(x + y) - 5(x - y)$

Simplify each expression.

34. $\frac{1}{2}\left(-\frac{6}{5}x - 40\right)$

35. $-\frac{3}{7}\left(14x - \frac{7}{9}\right)$

36. $18 - 16\left(x - \frac{3}{8}\right)$

37. $3(x + 2) - \frac{2}{5}(10x + 30)$

38. $\frac{5}{6}(12x + 6) + \frac{1}{2}(6x + 14)$

39. $-\frac{1}{2}(2x - 9) - \left(x + \frac{3}{2}\right)$

40. $\frac{2}{3}(9x + 15) - \frac{3}{4}(20x - 8)$

Answers to Section 1.3 Exercises

- | | |
|--|-------------------------------------|
| 1. 6 | 21. $2a - 9b + 3$ |
| 2. -60 | 22. $-4x - 2y$ |
| 3. -7 | 23. $\frac{11}{24}x + \frac{7}{20}$ |
| 4. $\frac{5}{11}$ | 24. $-\frac{13}{20}x + \frac{3}{5}$ |
| 5. 25 | 25. $48a - 24$ |
| 6. 4 | 26. $-3.9x - 4.68$ |
| 7. 10 | 27. $-5x + 15$ |
| 8. -4 | 28. $-12x + 39$ |
| 9. -5 | 29. $-7x + 13$ |
| 10. -49 | 30. $29x + 31$ |
| 11. 7 | 31. $5x - 88$ |
| 12. -22.5 | 32. $-11x + 5$ |
| 13. $5x + 8 + 3 = 5x + 11$ | 33. $-3x + 7y$ |
| 14. $9 \cdot 2 \cdot x = 18x$ | 34. $-\frac{3}{5}x - 20$ |
| 15. $6x + (2 + 7) = 6x + 9$ | 35. $-6x + \frac{1}{3}$ |
| 16. $\left(4 \cdot \frac{1}{4}\right) \cdot a = 1 \cdot a = a$ | 36. $-16x + 24$ |
| 17. $4x - 6$ | 37. $-x - 6$ |
| 18. $3x + 4y$ | 38. $13x + 12$ |
| 19. $-8x + 11$ | 39. $-2x + 3$ |
| 20. $xy - 2x$ | 40. $-9x + 16$ |

Mixed Review

Sections 1.1 – 1.3

Simplify.

1. $-15 \div |7 - 10| \times \sqrt{49}$

2. $\left(-\frac{5}{6}\right)\left(-\frac{8}{15}\right)$

3. $-8 \div \frac{22}{3}$

4. $\frac{\frac{24}{25}}{\frac{20}{15}}$

5. $\left(-\frac{3}{2}\right)^3$

6. $-\frac{3}{10} + \frac{5}{6}$

7. $-\frac{7}{15} - \frac{2}{3}$

8. $\frac{7}{3} - \frac{3}{2} \times \frac{10}{3}$

9. $\left(\frac{2}{3} + \frac{7}{2}\right) \div \left(-\frac{5}{4}\right)^2$

10. $\frac{12 + 2^3}{6 \cdot 4 - (-9 + 5)}$

Evaluate.

11. $3a - 8b$ if $a = \frac{5}{6}$ and $b = \frac{3}{10}$

12. $\left(\frac{5}{a}\right)^2 \div \left(\frac{7}{a} + \frac{b}{8}\right)$ if $a = 4$ and $b = 1$

Answers to Mixed Review

1. -35

2. $\frac{4}{9}$

3. $-\frac{12}{11}$

4. $\frac{18}{25}$

5. $-\frac{27}{8}$

6. $\frac{8}{15}$

7. $-\frac{17}{15}$

8. $-\frac{8}{3}$

9. $\frac{8}{3}$

10. $\frac{5}{7}$

11. $\frac{1}{10}$

12. $\frac{5}{6}$