

**CHAPTER**

**1**

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# **Real Numbers and Algebraic Expressions**

Section 1.1 Integers

Section 1.2 Rational Numbers

Section 1.3 Algebraic Expressions

*Fall 2019*

## Section 1.1 Objectives

- Recognize natural numbers, whole numbers, and integers.
- Determine the integer value of points on the number line.
- Plot integers on the number line.
- Evaluate the absolute value of integers.
- Add, subtract, multiply, and divide integers.
- Evaluate integers raised to a power.
- Evaluate roots of perfect squares, cubes, and fourths.
- Use order of operations to simplify arithmetic expressions that contain integers.

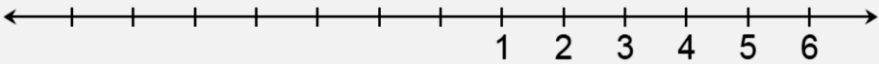
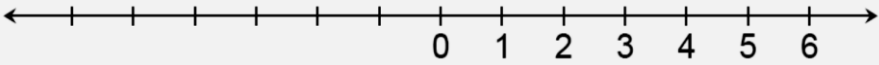
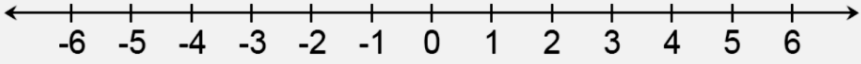
# SECTION 1.1 Integers

## INTRODUCTION

In this section, you will perform arithmetic operations with integers. But first, let's begin by defining the integers and other kinds of numbers that will be used in this chapter.

## NUMBER SYSTEMS

Numbers can be classified into different *number systems* (groups). Three number systems are described in the following chart.

NUMBER SYSTEMS	
<b>Natural Numbers</b>	<p>The <i>natural numbers</i> are also called the <i>counting numbers</i>.</p> <p>The <i>natural numbers</i> are <math>\{1, 2, 3, 4, 5, \dots\}</math>.</p> <p>Look at the <i>natural numbers</i> on the number line below.</p>  <p>Note: There is no greatest or “last” <i>natural number</i>. The arrow on the number line indicates that the numbers continue on endlessly.</p>
<b>Whole Numbers</b>	<p>The <i>whole numbers</i> include the set of <i>natural numbers</i> and the number 0.</p> <p>The <i>whole numbers</i> are <math>\{0, 1, 2, 3, 4, 5, \dots\}</math>.</p> <p>Look at the <i>whole numbers</i> on the number line below and notice that the number 0 is included.</p>  <p>Note: There is no greatest or “last” <i>whole number</i>. The arrow on the number line indicates that the numbers continue on endlessly.</p>
<b>Integers</b>	<p>The <i>integers</i> are the positive and negative <i>counting numbers</i> and the number 0.</p> <p>The <i>integers</i> are <math>\{\dots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \dots\}</math>.</p> <p>Look at the <i>integers</i> on the number line below.</p>  <p>Note: There is no least or “first” <i>integer</i> and there is no greatest or “last” <i>integer</i>. The arrows on the number line indicate that the numbers continue on endlessly.</p>

**REVIEW:** NUMBER SYSTEMS 

**PRACTICE:** Circle each number system that the given number belongs to, and cross out each number system that the given number does not belong to.

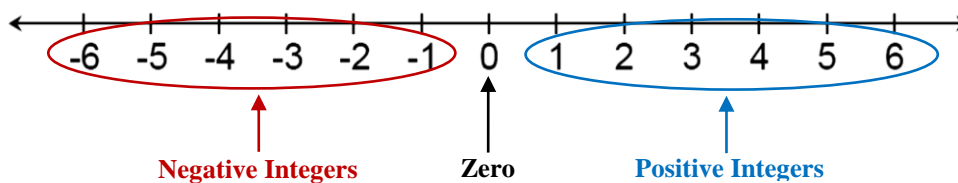
- |    |    |                 |               |          |
|----|----|-----------------|---------------|----------|
| 1. | 7  | Natural numbers | Whole numbers | Integers |
| 2. | -5 | Natural numbers | Whole numbers | Integers |
| 3. | -2 | Natural numbers | Whole numbers | Integers |
| 4. | 0  | Natural numbers | Whole numbers | Integers |

**Answers:** 

- |    |    |                            |                          |          |
|----|----|----------------------------|--------------------------|----------|
| 1. | 7  | Natural numbers            | Whole numbers            | Integers |
| 2. | -5 | <del>Natural numbers</del> | <del>Whole numbers</del> | Integers |
| 3. | -2 | <del>Natural numbers</del> | <del>Whole numbers</del> | Integers |
| 4. | 0  | <del>Natural numbers</del> | Whole numbers            | Integers |

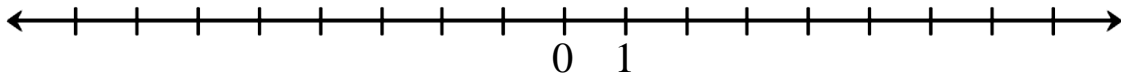
POSITIVE AND NEGATIVE INTEGERS	
The set of <i>integers</i> can be categorized into three distinct subsets (groups):	
<b>Positive</b> Integers	{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, ... }
Zero	{ 0 }
<b>Negative</b> Integers	{ ..., -11, -10, -9, -8, -7, -6, -5, -4, -3, -2, -1 }

Look at these subsets on the number line below.

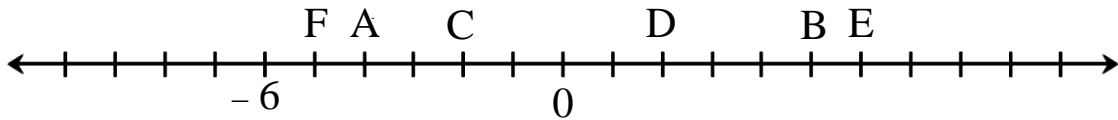


**PRACTICE:**


- Circle all the positive integers in the list:                    –                    –
- Circle all the negative integers in the list:                    –                    –
- Place each of the following numbers on the number line below. 2   5   –3   –1   6   –6  
(For a frame of reference, the numbers 0 and 1 are given.)




- What is the value of each of the points named by the capital letters on the number line below? (For a frame of reference, the numbers 0 and –6 are given.)

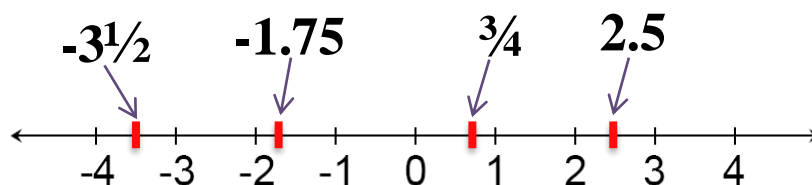


**Answers:**

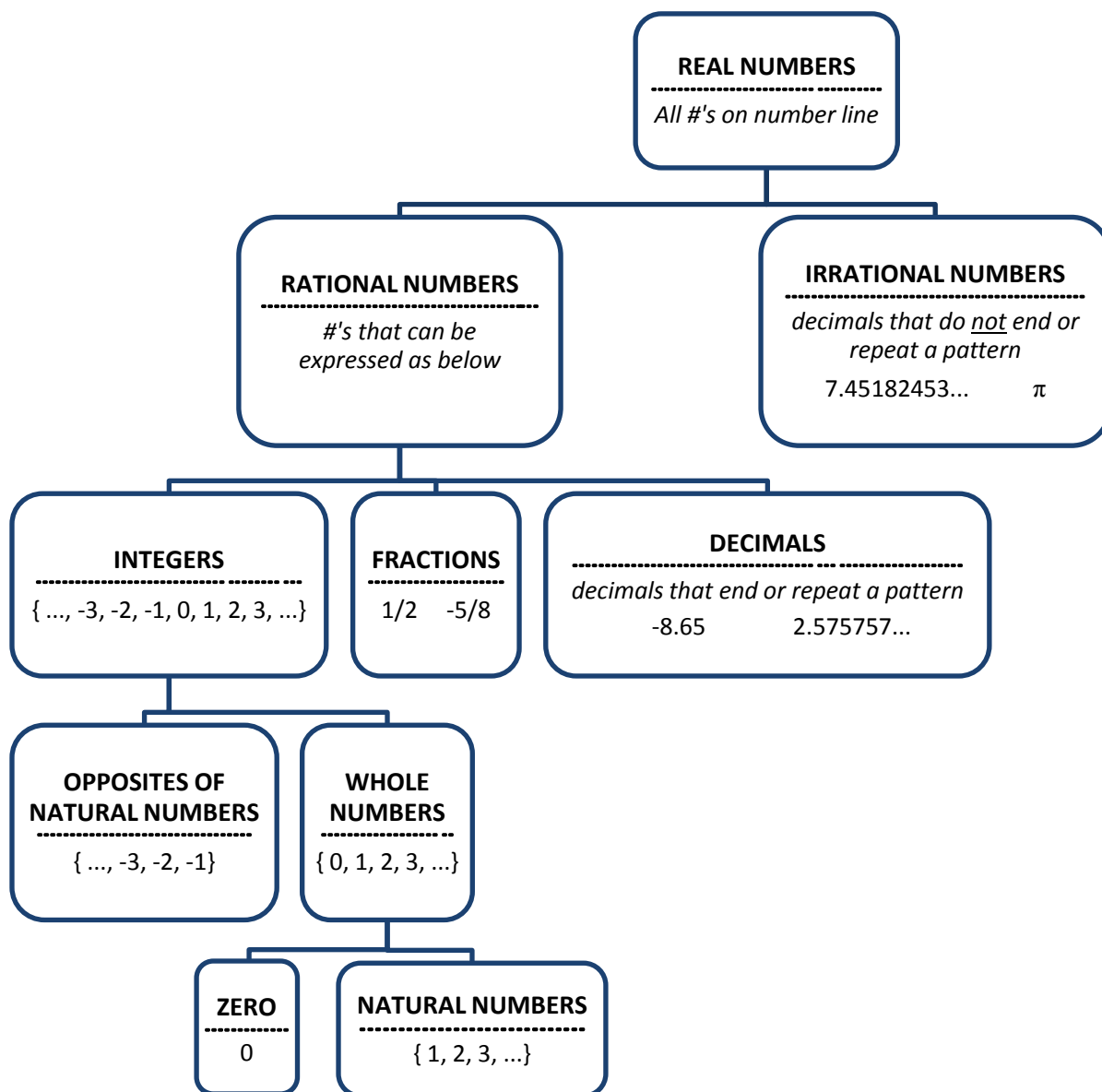
- –                    – ○
- ○ – ○
- 

B =                    =C 2                    D =                    E =                    F 

Before we continue our study of integers, we should point out that the number line is a *densely* populated line. The integers are few and far between, mathematically speaking. There are many more numbers on the number line than just the integers. For instance, there are fractions and decimals. Look at the location of some fractions and decimals on the number line below.



The fractions and decimals belong to a number system called the Rational Numbers. You will study these numbers later in this book. You will study other non-integers in future math courses. So, to help you become familiar with the various kinds of numbers used in mathematics, an overview of the number systems is presented below in diagram form. The diagram describes each number system and allows you to see the relationships among them.



Strangely Beautiful Fact: If you pick any real number (integer, fraction, decimal) on the number line, there is no definitive real number “right next to it” on either side. That is, there is no *closest* real number either to the right or the left. There are also no gaps or holes in the number line. Strange. Beautiful. Fact. This is why mathematicians say that the real number line is a *continuum*.

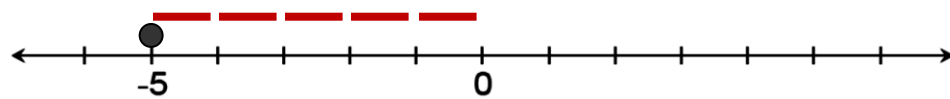
## ABSOLUTE VALUE

The **absolute value** of a real number  $r$  is denoted by the symbol  $|r|$ . For example, in mathematical notation, we write  $|3|$ . This is read, “The absolute value of 3.”

**Absolute value** can be defined as distance. The **absolute value** of a number is the distance from that number to 0 along the number line. Since distance is understood to be a non-negative measurement (a value greater than or equal to 0), the **absolute value** of a number is also greater than or equal to 0.

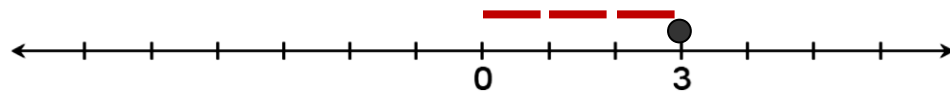
**EXAMPLES:** Evaluate each absolute value problem.

1.  $|-5|$



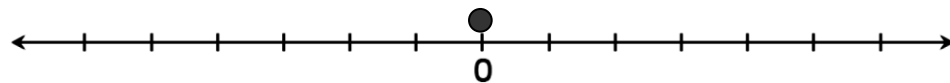
$-5$  is 5 units away from 0. Therefore,  $|-5| = 5$ . This is read, “The absolute value of  $-5$  is 5.”

2.  $|3|$



$3$  is 3 units away from 0. Therefore,  $|3| = 3$ . This is read, “The absolute value of 3 is 3.”

3.  $|0|$



$0$  is 0 units away from 0. Therefore,  $|0| = 0$ . This is read, “The absolute value of 0 is 0.”

The easiest way to evaluate absolute value problems is using the two facts below.

ABSOLUTE VALUE	
1. The <b>absolute value</b> of 0 is 0. 2. The <b>absolute value</b> of every other real number is <u>positive</u> .	Math Notation: $ r $

**EXAMPLES:** Evaluate each absolute value problem.

1.  $|6|$  The absolute value of the number is positive.  
 $|6| = 6$  The absolute value of 6 is 6.
2.  $|-6|$  The absolute value of the number is positive.  
 $|-6| = 6$  The absolute value of  $-6$  is 6.

**REVIEW:** ABSOLUTE VALUE**PRACTICE:** Evaluate.

1.  $|-7|$

4.  $|8|$

2.  $|4|$

5.  $|-9|$

3.  $|0|$

**Answers:**

1.



4.

2.



5.

3.



Next, we show some more examples of absolute value problems. Keep in mind that the absolute value symbol  $| |$  is treated as parentheses for the purpose of order of operations.

This means that operations inside the  $| |$  symbol must be performed before applying the definition of absolute value. Make sure that there is only one number inside the  $| |$  symbol before applying the definition of absolute value.

**EXAMPLES:** Evaluate each absolute value problem.

1.  $-|537|$

$$\begin{array}{l} -|537| \\ \downarrow \underbrace{\hspace{1.5cm}} \\ -537 \end{array}$$

There is only one number inside the  $| |$  symbol. So, we apply the definition of absolute value and evaluate  $|537|$ . This gives 537.

$$-537$$

The negative sign that was in front of the absolute value is applied by just bringing it down to the left of the number.

$$-537$$

The answer is  $-537$ .

2.  $-|-1|$

$$\begin{array}{l} -|-1| \\ \downarrow \underbrace{\hspace{1.5cm}} \\ -1 \end{array}$$

There is only one number inside the  $| |$  symbol. So, we apply the definition of absolute value and evaluate  $|-1|$ . This gives 1.

$$-1$$

The negative sign that was in front of the absolute value is applied by just bringing it down to the left of the number.

$$-1$$

The answer is  $-1$ .



3.  $|3+5|$

$|3+5|$

There is more than one number inside the  $|$   $|$ . So, we must perform the operation inside the absolute value first:  $3+5$  gives 8.

$|8|$

Now that there is only one number inside the  $|$   $|$ , we apply the definition of absolute value and evaluate  $|8|$ . This gives 8.

8

The answer is 8.

4.  $-|18-7|$

$-|18-7|$

There is more than one number inside the  $|$   $|$ . So, we must perform the operation inside the absolute value first:  $18-7$  gives 11.

$-|11|$

Now that there is only one number inside the  $|$   $|$ , we apply the definition of absolute value and evaluate  $|11|$ . This gives 11.

$-11$

The negative sign that was in front of the absolute value is applied by just bringing it down to the left of the number.

$-11$

The answer is  $-11$ .

**PRACTICE:** Evaluate.

1.  $-|21|$

5.  $-|38|$

2.  $-|-4|$

6.  $-|-62|$

3.  $|9-6|$

7.  $|20-4|$

4.  $-|7-5|$

8.  $-|7+2|$

**Answers:**

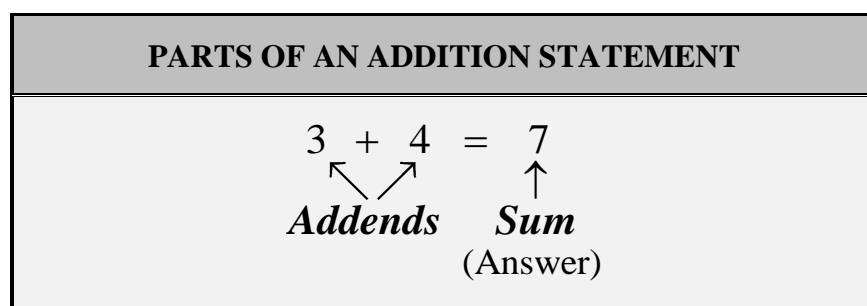


## ARITHMETIC OPERATIONS ON INTEGERS

Recall the set of numbers  $\{ \dots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \dots \}$  called the integers. Now we will begin to use the integers in arithmetic operations. We will review how to add, subtract, multiply, and divide with the integers.

### ADDITION OF INTEGERS

In an addition problem, the numbers that are being added are called *addends*, and the answer to the addition problem is called the *sum*.



It is important that you become familiar with the various ways that addition problems may be written. Sometimes you will see a problem written with one or both addends in parentheses. Other times, parentheses will not be used. With or without parentheses, either way is acceptable. Look at all the ways the same problem can be written.

$$-2 + -6$$

$$-2 + (-6)$$

$$(-2) + (-6)$$

Now it is time to learn the procedure for adding integers. We will show four different methods that can be used to add integers. The methods are listed below.

- (1) Number Line Method
- (2) Money Method
- (3) Signed Chip Method
- (4) SSS / DDD Method

You may be wondering if you will need to know and use all of the methods. The answer is “no.” It is important that you learn the SSS / DDD Method because this method will be necessary as the addition problems become more complex in this and future math courses. So, why are we presenting three other methods? The reason is that the other methods provide a more visual approach to the problems, making it easier to understand how we arrive at the answer. In other words, the three other methods may help you make sense of the SSS / DDD Method.

NUMBER LINE METHOD

Addition on the number line is shown by moving a specified number of units to the left or right.

**ADDITION OF INTEGERS: NUMBER LINE METHOD**

**First Number in Problem:** Start at this point on the number line.

**Second Number in Problem:** On the number line, move this many units from the starting point.

- If the second number is *positive*, move *right*.
- If the second number is *negative*, move *left*.

**Answer:** The point at which you stop on the number line.

**EXAMPLES:** Evaluate.

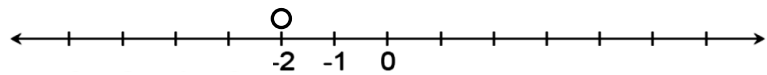
**Number Line Method**



1.  $-2 + -3$

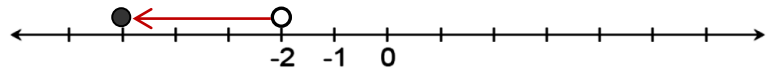
First Number:  $-2$

Place a mark at  $-2$  to show the starting point:



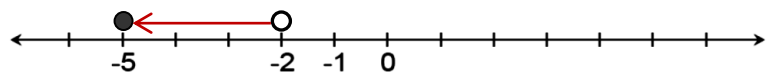
Second Number:  $-3$

Since we are adding a *negative* number ( $-3$ ), move 3 units to the *left*:



Answer:  $-5$

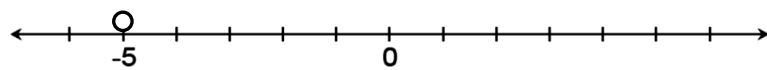
We stopped at the number  $-5$ . Therefore,  $-2 + -3 = -5$



2.  $-5 + 2$

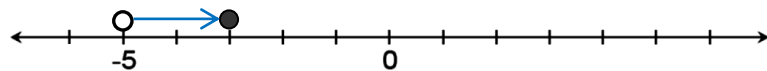
First Number:  $-5$

Place a mark at  $-5$  to show the starting point:



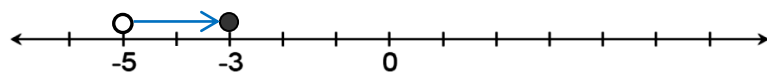
Second Number:  $2$

Since we are adding a *positive* number ( $2$ ), move 2 units to the *right*:



Answer:  $-3$

We stopped at the number  $-3$ . Therefore,  $-5 + 2 = -3$ .

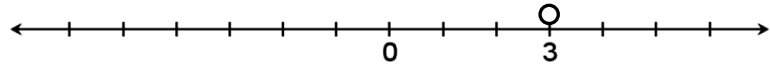
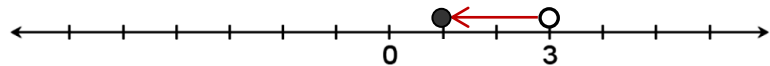


3.

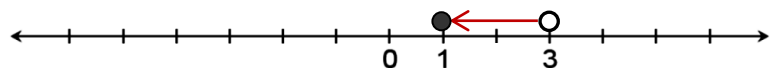
3.  $3 + (-2)$

First Number: 3

Place a mark at 3 to show the starting point:

Second Number:  $-2$ Since we are adding a *negative* number ( $-2$ ), move 2 units to the *left*:

Answer: 1

We stopped at the number 1. Therefore,  $3 + (-2) = 1$ **PRACTICE:** Evaluate.

1.  $-5 + -1$

4.  $-2 + (-7)$

2.  $-5 + 3$

5.  $-3 + 8$

3.  $6 + (-4)$

6.  $1 + -9$

**Answers:****MONEY METHOD**

This method involves thinking of integers as amounts of money either received or spent.

**ADDITION OF INTEGERS – MONEY METHOD****Addends**

- *Positive* integers represent money you *have* or *receive*.
- *Negative* integers represent money you *owe* or *spend*.

**Answer**

- *Add* the money you *have* or *receive*.
- *Subtract* the money you *owe* or *spend*.

**Sign of Answer**

- If you *owe* money, the answer is *negative*.
- If you *have* money, the answer is *positive*.

**EXAMPLES:** Evaluate.**Money Method****\$\$\$**

1.  $-4 + (-2)$

The negative number  $(-4)$  indicates that you lost 4 dollars to your Uncle Larry during one hand of a card game. So, you are down 4 dollars.

The negative number  $(-2)$  being added indicates that you then lost 2 more dollars to Uncle Larry in the next hand of that card game.

Where do you stand with Uncle Larry now? (*Hopefully, at least 50 feet away!*)

Now you owe your uncle 6 dollars. You're now 6 dollars in the hole.

We indicate being in the hole or in debt with a negative number.

Therefore,  $-4 + (-2) = -6$ .

*I should have warned you about Uncle Larry. He cheats at cards!*

2.  $9 + (-6)$

The next time you see Uncle Larry you have 9 dollars in your pocket.

In the addition problem, the positive number  $(9)$  indicates that you have 9 dollars.

The negative number  $(-6)$  being added indicates that you gave your Uncle Larry 6 dollars.

Where do you stand with your uncle? You paid him the 6 dollars you owed him. You now have 3 dollars left in your pocket.

We indicate that you have money with a positive number.

Therefore,  $9 + (-6) = 3$ .

*Don't you feel relieved not to owe Uncle Larry any money?*

**PRACTICE:** Evaluate.

1.  $-3 + (-1)$

4.  $(-5) + (-9)$

2.  $6 + (-4)$

5.  $3 + (-8)$

3.  $-2 + 7$





6.  $-10 + 4$

**Answers:**

SIGNED CHIP METHOD

This method represents integers as + or - chips. Addition is shown by combining all the chips and pairing + and - chips to equal 0. The sum is represented by the chips that remain.

**ADDITION OF INTEGERS – SIGNED CHIP METHOD**

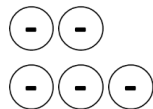
1. Draw positive chips  to represent the positive integers in the problem.
2. Draw negative chips  to represent the negative integers in the problem.
3. Cross off pairs of  and  chips until only one type of chip remains. Each pair of positive and negative chips is equal to zero.
4. The answer is the number of chips left, and the sign is the same as those chips.

**EXAMPLES:** Evaluate.

**Signed Chip Method**



1.  $-2 + -3$



Draw 2 negative chips to represent  $-2$ .

Draw 3 negative chips to represent  $-3$ .

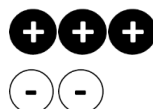


To add, we combine all the chips.

We have 5 negative chips.

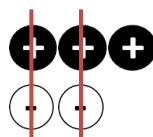
Therefore,  $-2 + -3 = -5$ .

2.  $3 + (-2)$



Draw 3 positive chips to represent 3.

Draw 2 negative chips to represent  $-2$ .



Cross off pairs of positive and negative chips.

Each pair equals 0.

What is left? One positive chip.

Therefore,  $3 + (-2) = 1$

**PRACTICE:** Evaluate.

1.  $-5 + -1$

2.  $6 + (-4)$

3.  $-4 + 9$

4.  $(-6) + (-8)$

5.  $2 + -7$

6.  $-6 + 1$

**Answers:**

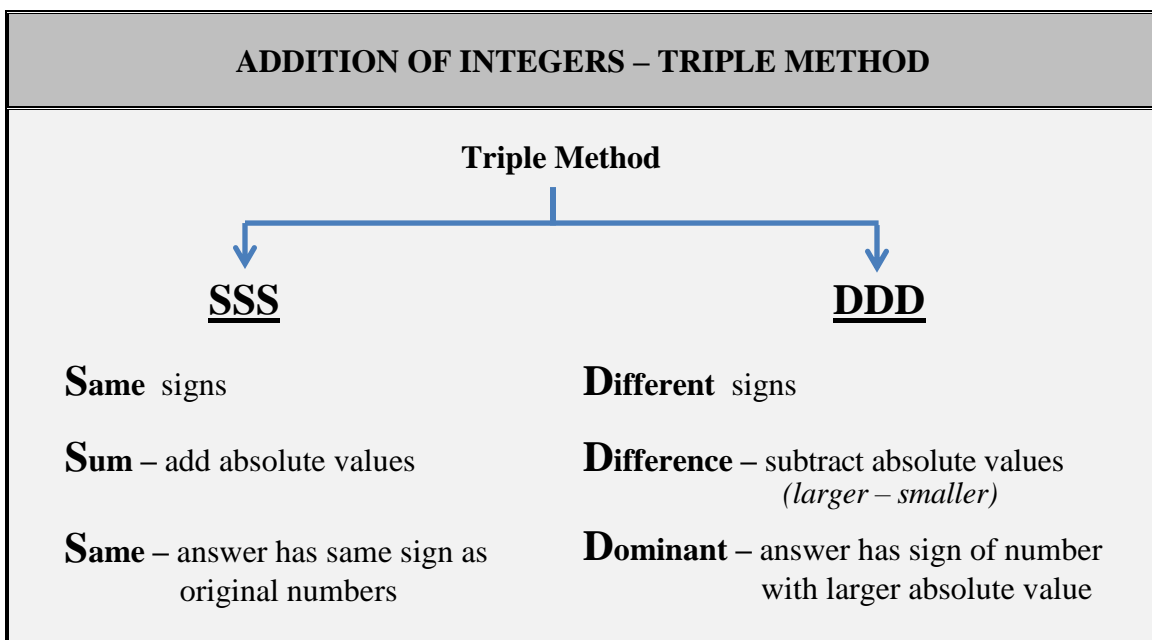


### TRIPLE METHOD

The previous three methods were presented to help you understand what it means to add positive and negative numbers. But it is not practical to represent and evaluate more involved problems like  $-23 + 47$  on a number line or with chips.

So, now you will learn one last method for adding integers. It is called the Triple Method. This method is the most important method to learn since it will be necessary for addition problems that are more complex. There are actually two different versions of this method. One is named SSS, and the other is named DDD. These 3-letter names are why we call this the “triple” method. We decide whether to use the SSS or DDD version based on the signs of the numbers in the addition problem.

- **SSS** – used if the numbers in the problem have the same sign
- **DDD** – used if the numbers in the problem have different signs



**EXAMPLES:** Evaluate.**Triple Method****SSS or DDD**

1.  $-3 + (-4)$

**Same:** We are adding two numbers that have the same sign. $-3$  and  $-4$  are both negative.**Sum:** Compute the sum of the absolute values of the numbers.

$$|-3| = 3 \quad |-4| = 4$$

First, determine the absolute value of each number.

$$3 + 4 = 7$$

Next, compute the sum.This means to add the absolute values.**Same:** The final answer takes the same sign as the original numbers in the problem.The numbers in the original problem were  $-3$  and  $-4$ .

Since those numbers were negative, our final answer is negative.

Therefore,  $-3 + (-4) = -7$ .

2.  $-6 + 4$

**Different:** We are adding two numbers that have different signs.The number  $-6$  is a negative number, and the number  $4$  is a positive number.**Difference:** Compute the difference between the absolute values of the two numbers.

$$|-6| = 6 \quad |4| = 4$$

First, determine the absolute value of each number.

$$6 - 4 = 2$$

Next, compute the difference of the absolute values.This means to subtract: *Larger Number – Smaller Number***Dominant:** The final answer takes the sign of the dominant number.The dominant number is the one with the larger absolute value.The numbers in the original problem are  $-6$  and  $4$ .The dominant number is  $-6$  since it has the larger absolute value.

Since the dominant number is negative, the final answer is also negative.

Therefore,  $-6 + 4 = -2$ .**REVIEW:** ADDING INTEGERS – TRIPLE METHOD



**PRACTICE:** Evaluate.

1.  $-5 + -1$

2.  $-3 + (-1)$

3.  $-5 + 3$

4.  $6 + (-4)$

5.  $-1 + 5$

6.  $-8 + -2$

7.  $4 + -10$

8.  $-7 + (-9)$

**Answers:**



## SUBTRACTION OF INTEGERS

At all levels of mathematics, one key strategy to solving a problem is to use what you already know – to build logically on ideas that you have already learned. This will be our approach to subtracting integers. We will change a subtraction problem into an equivalent addition problem. Then we will simply use the methods that we already learned for adding two integers.

To rewrite subtraction problems as equivalent addition problems, we use a method called **Adding the Opposite**. Let's explain the procedure. Keep the first number in the subtraction problem the same. Change the operation from subtraction to addition. Change the second number to its opposite. This means we change the sign of the second number. If it is positive, we change it to negative. If it is negative, we change it to positive.

SUBTRACTION OF INTEGERS (ADDING THE OPPOSITE)		
<ul style="list-style-type: none"> <li>• KEEP the first number the same.</li> <li>• CHANGE the operation from subtraction to addition.</li> <li>• CHANGE the second number to its opposite.</li> </ul>		
Subtracting a Positive	Subtracting a Negative	
$  \begin{array}{ccc}  a & - & b \\  \text{(KEEP)} & \text{(CHANGE)} & \text{(CHANGE)} \\  \downarrow & \downarrow & \downarrow \\  a & + & -b \\  \text{So, } a - b = a + (-b).  \end{array}  $	$  \begin{array}{ccc}  a & - & (-b) \\  \text{(KEEP)} & \text{(CHANGE)} & \text{(CHANGE)} \\  \downarrow & \downarrow & \downarrow \\  a & + & b \\  \text{So, } a - (-b) = a + b.  \end{array}  $	

**EXAMPLES:** Evaluate.

1.  $-2-1$

First, rewrite the subtraction problem as an addition problem by adding the opposite.

Adding the Opposite		
-2	-	1
(KEEP)	(CHANGE)	(CHANGE)
↓	↓	↓
-2	+	-1

First Number: KEEP  $-2$  the same.

Operation: CHANGE subtraction to addition.

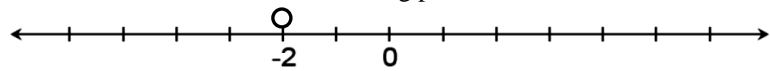
Second Number: CHANGE 1 to its opposite which is  $-1$ .

So, the addition problem is  $-2 + -1$ .

Next, evaluate the addition problem  $-2 + -1$  using one of the addition methods you already learned. Let's use the **Number Line Method**.

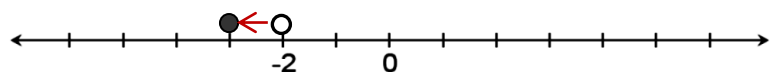
First Number:  $-2$

Place a mark at  $-2$  to show the starting point:



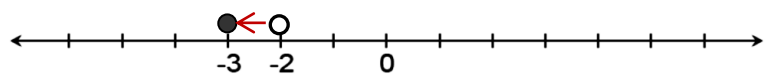
Second Number:  $-1$

Since we are adding a negative number ( $-1$ ), move 1 unit to the left:



Answer:  $-3$

We stopped at the number  $-3$ . Therefore,  $-2 + -1 = -3$ .



Therefore,  $-2-1 = -2 + -1 = -3$ .

2.  $3-5$

First, rewrite the subtraction problem as an addition problem by adding the opposite.

Adding the Opposite		
3	-	5
(KEEP)	(CHANGE)	(CHANGE)
↓	↓	↓
3	+	-5

First Number: KEEP 3 the same.

Operation: CHANGE subtraction to addition.

Second number: CHANGE 5 to its opposite which is  $-5$ .

So, the addition problem is  $3 + -5$ .

Next, evaluate the addition problem  $3 + -5$  using one of the addition methods you already learned. Let's go with the **Money Method** this time.

We will use a bank account to help us understand the problem.

The positive number (3) indicates that you have 3 dollars in your account.

The negative number ( $-5$ ) indicates that you spent 5 dollars, by writing a check or using your debit card.

What is the balance in your account? You spent 2 dollars more than you had, so you are "overdrawn".

We indicate being "overdrawn" with a negative number ( $-2$ ).

Therefore,  $3-5 = 3 + (-5) = -2$ .

3.  $-5 - (-7)$

First, rewrite the subtraction problem as an addition problem by adding the opposite.

Adding the Opposite		
-5	-	-7
(KEEP)	(CHANGE)	(CHANGE)
↓	↓	↓
-5	+	7

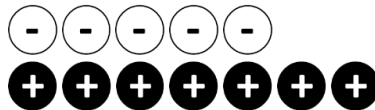
First Number: KEEP  $-5$  the same.

Operation: CHANGE subtraction to addition.

Second Number: CHANGE  $-7$  to its opposite which is  $7$ .

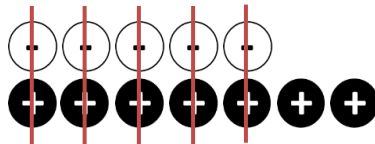
So, the addition problem is  $-5 + 7$ .

Next, evaluate the addition problem  $-5 + 7$  using one of the addition methods you already learned. For this problem we will use the *Signed Chip Method*.



Draw 5 negative chips to represent  $-5$ .

Draw 7 positive chips to represent  $7$ .



Cross off pairs of positive and negative chips. Each pair equals 0.

What is left? Two positive chips.

Therefore,  $-5 - (-7) = -5 + 7 = 2$ .

4.  $-6 - 8$

First, rewrite the subtraction problem as an addition problem by adding the opposite.

Adding the Opposite		
-6	-	8
(KEEP)	(CHANGE)	(CHANGE)
↓	↓	↓
-6	+	-8

First Number: KEEP  $-6$  the same.

Operation: CHANGE subtraction to addition.

Second Number: CHANGE  $8$  to its opposite which is  $-8$ .

So, the addition problem is  $-6 + -8$ .

Next, we evaluate the addition problem  $-6 + -8$  using the *Triple Method (SSS)*.

Same: The numbers in the addition problem have the same sign (both negative).

Sum: Add the absolute values of the numbers  $\rightarrow 6 + 8 = 14$

Same: The answer has the same sign as the numbers in the addition problem (negative).

Therefore,  $-6 - 8 = -6 + -8 = -14$

5.  $4 - 9$

Rewrite the subtraction problem as an addition problem by adding the opposite.

Adding the Opposite		
4	-	9
(KEEP)	(CHANGE)	(CHANGE)
↓	↓	↓
4	+	-9

Evaluate the addition problem  $4 + -9$ . We use the *Triple Method (DDD)*.

**Different:** The numbers in the addition problem have different signs.

**Difference:** Subtract the absolute values of the numbers  $\rightarrow 9 - 4 = 5$

**Dominant:**  $-9$  has the larger absolute value. Since it is negative, the answer is negative.

Therefore,  $4 - 9 = 4 + -9 = -5$ .

6.  $9 - (-3)$

Rewrite the subtraction problem as an addition problem by adding the opposite.

Adding the Opposite		
9	-	-3
(KEEP)	(CHANGE)	(CHANGE)
↓	↓	↓
9	+	3

Therefore,  $9 - (-3) = 9 + 3 = 12$ .

7.  $-3 - (-7)$

$$\begin{aligned} & -3 - (-7) \\ = & -3 + 7 \\ = & 4 \end{aligned}$$

Rewrite as an addition problem by adding the opposite.

Add using DDD.

8.  $-5 - 5$

$$\begin{aligned} & -5 - 5 \\ = & -5 + -5 \\ = & -10 \end{aligned}$$

Rewrite as an addition problem by adding the opposite.

Add using SSS.

**REVIEW:** SUBTRACTING INTEGERS **PRACTICE:** Evaluate.

1.  $-4-3$

2.  $4-7$

3.  $-3-(-5)$

4.  $3-(-3)$

5.  $-4-(-7)$

6.  $-2-9$

7.  $8-10$

8.  $6-(-1)$

9.  $-9-(-7)$

10.  $-5-8$

**Answers:****REVIEW:** ADDING INTEGERS – TRIPLE METHOD **REVIEW:** SUBTRACTING INTEGERS **MIXED PRACTICE:** Evaluate.

1.  $-4+-6$

2.  $-7+3$

3.  $-1+8$

4.  $5-(-2)$

5.  $3-10$

6.  $-6-3$

7.  $-8+(-2)$

8.  $4+(-6)$

9.  $-8+8$

10.  $9+(-1)$

11.  $-7-1$

12.  $4-13$

13.  $-5-(-11)$

14.  $12-(-3)$

**Answers:**

## MULTIPLICATION AND DIVISION OF INTEGERS

We begin with a review of some basic terminology associated with multiplication and division. The boxes below show the names of each part of a multiplication problem and each part of a division problem. Make sure you are familiar with the names shown.

PARTS OF A MULTIPLICATION STATEMENT	
$3 \times 4 = 12$	$\uparrow$ <b>Product</b> (Answer)

PARTS OF A DIVISION STATEMENT	
$8 \div 2 = 4$	$\uparrow$ <b>Quotient</b> (Answer)

Next, we review the mathematical notation used to represent multiplication and division. The boxes below show how these two operations can be expressed using different types of notation. You should be familiar with all the ways to represent multiplication and division.

MULTIPLICATION NOTATION		
$3 \times -4$	$3 \cdot -4$	
$3 \times (-4)$	$3 \cdot (-4)$	$3(-4)$
$(3) \times (-4)$	$(3) \cdot (-4)$	$(3)(-4)$

DIVISION NOTATION		
$8 \div 2 = 4$	$\frac{8}{2} = 4$	$2 \overline{)8}$

Now we review how to multiply and divide with positive and negative numbers. There are just two rules to remember. To multiply or divide two nonzero real numbers:

1. If the numbers have the same sign, the answer is positive.
2. If the numbers have different signs, the answer is negative.

MULTIPLICATION AND DIVISION OF INTEGERS			
<b>Same Signs</b> $\downarrow$ <b>Positive Answer</b>	<b>Multiplication</b> $\oplus \times \oplus = \oplus$ $\ominus \times \ominus = \oplus$	<b>Division</b> $\oplus \div \oplus = \oplus$ $\ominus \div \ominus = \oplus$	
<b>Different Signs</b> $\downarrow$ <b>Negative Answer</b>	<b>Multiplication</b> $\oplus \times \ominus = \ominus$ $\ominus \times \oplus = \ominus$	<b>Division</b> $\oplus \div \ominus = \ominus$ $\ominus \div \oplus = \ominus$	

**EXAMPLES:** Evaluate.

1.  $(-2)(-3)$

$(-2)(-3) = 6$

Same Signs (both  $\ominus$ )  $\rightarrow$  Positive Answer

2.  $4 \cdot (-3)$

$4 \cdot (-3) = -12$

Different Signs (one  $\oplus$ , other  $\ominus$ )  $\rightarrow$  Negative Answer

3.  $(-12) \div (-3)$

$(-12) \div (-3) = 4$

Same Signs (both  $\ominus$ )  $\rightarrow$  Positive Answer

4.  $(-24) \div 8$

$(-24) \div 8 = -3$

Different Signs (one  $\oplus$ , other  $\ominus$ )  $\rightarrow$  Negative Answer**REVIEW:** MULTIPLYING INTEGERS**REVIEW:** DIVIDING INTEGERS**PRACTICE:** Evaluate.

1.  $4 \times (-6)$

2.  $(-5)(-7)$

3.  $3 \cdot 5$

4.  $(-6) \times 2$

5.  $8 \times (-8)$

6.  $36 \div (-6)$

7.  $(-49) \div (-7)$

8.  $(-18) \div 3$

9.  $32 \div 4$

10.  $(-45) \div (-5)$

**Answers:**

The multiplication and division rules that you just learned applied to nonzero numbers. Now we will consider how to multiply and divide when zero is one of the numbers in the problem. The rules for those types of problems are given below.

MULTIPLICATION INVOLVING ZERO		
$n \cdot 0 = 0$	$0 \cdot n = 0$	The product of a number and 0 is 0.
DIVISION INVOLVING ZERO		
$0 \div n = 0$	$\frac{0}{n} = 0$	If 0 is divided by any number (except 0), the <u>answer is 0</u> .
$n \div 0 = \text{undefined}$	$\frac{n}{0} = \text{undefined}$	If any number is divided by 0, the <u>answer is undefined</u> . In other words, there is no numerical answer.

**EXAMPLES:** Evaluate.

- $0 \cdot 6$   
 $0 \cdot 6 = 0$       The product of any number and 0 is 0.
- $7(0)$   
 $7(0) = 0$       The product of any number and 0 is 0.
- $0 \div (-5)$   
 $0 \div (-5) = 0$       Zero divided by any number is 0.
- $4 \div 0$   
 $4 \div 0 = \text{Undefined}$       Any number divided by zero is undefined.

**REVIEW:** MULTIPLICATION INVOLVING ZERO



**REVIEW:** DIVISION INVOLVING ZERO



**PRACTICE:** Evaluate.

- $2 \times 0$
- $0(-9)$
- $-8 \cdot 0$
- $(-4) \div 0$
- $0 \div 37$
- $5 \div 0$

**Answers:**





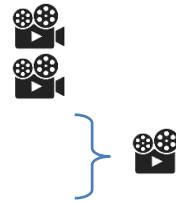
**REVIEW:** MULTIPLYING AND DIVIDING INTEGERS 

**REVIEW:** MULTIPLICATION AND DIVISION INVOLVING ZERO 

**MIXED PRACTICE:** Evaluate.

- |                       |                      |
|-----------------------|----------------------|
| 1. $(-4)(-5)$         | 9. $(-21) \div (-3)$ |
| 2. $-7 \cdot 8$       | 10. $54 \div (-6)$   |
| 3. $12 \cdot 0$       | 11. $0 \div 2$       |
| 4. $(-4) \times (-2)$ | 12. $2 \div 0$       |
| 5. $(7)(-3)$          | 13. $-30 \div 5$     |
| 6. $-6 \times 9$      | 14. $-48 \div (-4)$  |
| 7. $0 \cdot 5$        | 15. $6 \div 0$       |
| 8. $(-4)(0)$          | 16. $0 \div (-9)$    |

**Answers:**



**EXPONENTS**

You have reviewed how to add, subtract, multiply, and divide integers. Now you will study another operation – raising an integer to a power. An example is  $5^3$ . The number 3 is called the **exponent** (or power), and the number 5 is called the **base**. The exponent 3 tells us to perform the repeated multiplication  $5 \times 5 \times 5$ . The exponent 3 specifies the number of times the base 5 is used when multiplying.

<b>EXPONENTIAL NOTATION</b>	
Base $\rightarrow a^n \leftarrow$ Exponent  Indicates repeated multiplication	<b>Base</b> – the number being multiplied  <b>Exponent</b> – the number of times the base is used in the multiplication

<b>ZERO POWER PROPERTY</b>	
$a^0 = 1$	Any number (except 0) raised to the 0 power is 1.

**EXAMPLES:** Evaluate.

- 1.
- $2^4$
- This is read, “Two to the fourth power.”

$$\begin{aligned} & 2^4 && \text{The base is 2 and the **exponent is 4**.} \\ & = 2 \times 2 \times 2 \times 2 && \text{Multiply 2 four times.} \\ & = 16 && \text{The answer is 16.} \end{aligned}$$

- 2.
- $4^3$
- This is read, “Four to the third power” or “Four cubed.”

$$\begin{aligned} & 4^3 && \text{The base is 4 and the **exponent is 3**.} \\ & = 4 \cdot 4 \cdot 4 && \text{Multiply 4 three times.} \\ & = 64 && \text{The answer is 64.} \end{aligned}$$

- 3.
- $1^5$
- This is read, “One to the fifth power.”

$$\begin{aligned} & 1^5 && \text{The base is 1 and the **exponent is 5**.} \\ & = (1)(1)(1)(1)(1) && \text{Multiply 1 five times.} \\ & = 1 && \text{The answer is 1.} \end{aligned}$$

- 4.
- $-3^2$
- This is read, “The opposite of 3 to the second power” or “The opposite of 3 squared.”

This problem is different than the previous ones because it has a negative sign in front. However, we do the problem the same way as the previous problems. Just remember to **keep the negative sign in front**. Let’s also be attentive to the base.

$$\begin{aligned} & -3^2 && \text{The base is 3 and the **exponent is 2**.} \\ & = -(3)(3) && \text{Multiply 3 two times. This gives 9. Keep the negative sign in front.} \\ & = -9 && \text{The answer is } -9. \end{aligned}$$

- 5.
- $(-3)^2$
- This is read, “Negative 3 to the second power” or “Negative 3 squared.”

This problem is different than the last because it has a number in parentheses. **The number in parentheses is the base.** The exponent is outside the parentheses.

$$\begin{aligned} & (-3)^2 && \text{The base is } -3 \text{ since it is in parentheses. The **exponent is 2**.} \\ & = (-3)(-3) && \text{Multiply } -3 \text{ two times. This gives 9.} \\ & = 9 && \text{The answer is 9.} \end{aligned}$$

- 6.
- $-5^3$
- This is read, “The opposite of 5 to the third power” or “The opposite of 5 cubed.”

$$\begin{aligned} & -5^3 && \text{The base is 5 and the **exponent is 3**.} \\ & = -(5)(5)(5) && \text{Multiply 5 three times. This gives 125. Keep the negative sign in front.} \\ & = -125 && \text{The answer is } -125. \end{aligned}$$

7.  $(-5)^3$  This is read, “*Negative 5 to the third power*” or “*Negative 5 cubed*.”

$$\begin{aligned} & (-5)^3 && \text{The base is } -5 \text{ since it is in parentheses. The **exponent is 3.** \\ = & (-5)(-5)(-5) && \text{Multiply } -5 \text{ three times. This gives } -125. \text{} \\ = & -125 && \text{The answer is } -125. \end{aligned}$$

8.  $12^1$  This is read, “*Twelve to the first power*.”

$$\begin{aligned} & 12^1 && \text{The base is 12 and the exponent is 1.} \\ = & 12 && \text{Since the exponent is 1, there is only one 12.} \end{aligned}$$

9.  $0^5$  This is read, “*Zero to the fifth power*.”

$$\begin{aligned} & 0^5 && \text{The base is 0 and the exponent is 5.} \\ = & 0 \times 0 \times 0 \times 0 \times 0 && \text{Multiply 0 five times.} \\ = & 0 && \text{Remember that the product of any number and 0 is 0.} \end{aligned}$$

10.  $6^0$  This is read, “*Six to the zero power*.”

$$\begin{aligned} & 6^0 && \text{The base is 6 and the exponent is 0.} \\ = & 1 && \text{Any number (except for 0) raised to the 0 power is 1.} \end{aligned}$$

**REVIEW:** EXPONENTS



**REVIEW:** ZERO POWER



**PRACTICE:** Evaluate.

- |             |              |              |
|-------------|--------------|--------------|
| 1. $3^4$    | 6. $2^3$     | 11. $10^4$   |
| 2. $45^1$   | 7. $1^4$     | 12. $6^1$    |
| 3. $5^0$    | 8. $0^4$     | 13. $4^0$    |
| 4. $-4^2$   | 9. $-2^3$    | 14. $-2^4$   |
| 5. $(-4)^2$ | 10. $(-2)^3$ | 15. $(-2)^4$ |

**Answers:**



## ROOTS

An operation related to exponents is taking the **root** of a number, for example  $\sqrt{81}$  or  $\sqrt[3]{27}$ . The information in the box below reviews basic vocabulary and explains the meaning of the notation.

ROOT NOTATION	
Radical Sign	
↓	
Index →	$\sqrt[n]{a}$ ← Radicand
$\sqrt[n]{a}$ means the number that when raised to the power $n$ , gives $a$ .	
In other notation, $\boxed{?}^n = a$	
<i>Note: When there is no index shown, it is understood to be 2.</i>	

You should try to become familiar with the perfect squares, cubes, and fourths that are listed in the chart below. This will help you to simplify roots.

<b>Perfect Squares</b>	$1^2 = 1$	$2^2 = 4$	$3^2 = 9$	$4^2 = 16$	$5^2 = 25$
	$6^2 = 36$	$7^2 = 49$	$8^2 = 64$	$9^2 = 81$	$10^2 = 100$
<b>Perfect Cubes</b>	$1^3 = 1$	$2^3 = 8$	$3^3 = 27$	$4^3 = 64$	$5^3 = 125$
<b>Perfect Fourths</b>	$1^4 = 1$	$2^4 = 16$	$3^4 = 81$		

**EXAMPLES:** Evaluate.

1.  $\sqrt{9}$  This is read, “*The square root of 9.*”

$$\sqrt{9} = \sqrt[2]{9} \quad \text{Since no index is shown, it is understood to be 2.}$$

$$\boxed{\phantom{0}}^2 = 9 \quad \text{We need to know what number can be raised to the 2<sup>nd</sup> power to get 9.}$$

What number would you put in the box?

$$\boxed{3}^2 = 9 \quad \text{The answer is 3 because } 3^2 = 3 \cdot 3 = 9.$$

So,  $\sqrt{9} = 3$ .

2.  $\sqrt[3]{125}$  This is read, "The cube root of 125."

<sup>3</sup> = 125 We need to know what number can be raised to the 3<sup>rd</sup> power to get 125.  
What number would you put in the box?

<sup>3</sup> = 125 The answer is 5 because  $5^3 = 5 \times 5 \times 5 = 125$ .

So,  $\sqrt[3]{125} = 5$ .

3.  $\sqrt{169}$  This is read, "The square root of 169."

$\sqrt{169} = \sqrt[2]{169}$  Since no index is shown, it is understood to be 2.

<sup>2</sup> = 169 We need to know what number can be raised to the 2<sup>nd</sup> power to get 169.  
What number would you put in the box?

<sup>2</sup> = 169 The answer is 13 because  $13^2 = 13 \cdot 13 = 169$ .

So,  $\sqrt{169} = 13$ .

4.  $\sqrt[4]{81}$  This is read, "The fourth root of 81."

<sup>4</sup> = 81 We need to know what number can be raised to the 4<sup>th</sup> power to get 81.  
What number would you put in the box?

<sup>4</sup> = 81 The answer is 3 because  $3^4 = 3 \times 3 \times 3 \times 3 = 81$ .

So,  $\sqrt[4]{81} = 3$ .

### **REVIEW:** ROOTS



### **PRACTICE:** Evaluate.

1.  $\sqrt{16}$

4.  $\sqrt[4]{256}$

2.  $\sqrt[3]{27}$

5.  $\sqrt{25}$

3.  $\sqrt{225}$

6.  $\sqrt[3]{8}$

### **Answers:**




## ORDER OF OPERATIONS

Now you know how to perform all the operations – addition, subtraction, multiplication, division, exponents, and roots. But what if you have a problem that contains more than one operation? For instance, what if you have the problem  $-8^2 - 40 \div 2^3 \cdot (6 + 9)$ ? What operation should be performed first, second, third, etc.? It is important for everyone to do the problem the same way in order to get the same answer.

For this reason, mathematicians developed a set of rules for evaluating problems that involve more than one arithmetic operation. The rules, called the ***Order of Operations***, specify the order in which the computations should be performed. The Order of Operations is given below. It is important to follow these rules, one step at a time, in the order in which they are presented.

<b>ORDER OF OPERATIONS</b>	
<b>Step 1: Parentheses</b>	If there are any operations in parentheses, those computations should be performed first.
<b>Step 2: Exponents and Roots</b>	Simplify any numbers being raised to a power and any numbers under the $\sqrt{\quad}$ symbol.
<b>Step 3: Multiplication and Division</b>	Do these two operations in the order in which they appear, working from <i>left to right</i> .
<b>Step 4: Addition and Subtraction</b>	Do these two operations in the order in which they appear, working from <i>left to right</i> .

To help remember the Order of Operations, try using the phrase in the box on the left below.

<b>P</b> lease <b>E</b> xcuse <b>M</b> y <b>D</b> ear <b>A</b> unt <b>S</b> ally		<b>P</b> arentheses <b>E</b> xponents <i>and</i> <b>R</b> oots <b>M</b> ultiplication <i>and</i> <b>D</b> ivision ... <i>working from left to right</i> <b>A</b> ddition <i>and</i> <b>S</b> ubtraction ... <i>working from left to right</i>
---	---	--

**EXAMPLES:** Evaluate.

1.  $5 + 3^2 \times 4$

$$\begin{aligned}
 & 5 + 3^2 \times 4 \\
 &= 5 + \underbrace{3^2}_{9} \times 4 \\
 &= 5 + \underbrace{9 \times 4}_{36} \\
 &= 5 + 36 \\
 &= 41
 \end{aligned}$$

**Step 1: Parentheses.** There are none.**Step 2: Exponents and Roots.** We have an exponent. Evaluate  $3^2$  and get 9.**Step 3: Multiplication and Division.** Multiply  $9 \times 4$  and get 36.**Step 4: Addition and Subtraction.** Add  $5 + 36$  and get 41.**Answer:** The answer is 41.

2.  $\sqrt{9} - 8 - 5 + 2^2$

$$\begin{aligned}
 & \sqrt{9} - 8 - 5 + 2^2 \\
 &= \underbrace{\sqrt{9}}_3 - 8 - 5 + \underbrace{2^2}_4 \\
 &= 3 - 8 - 5 + 4 \\
 &= \underbrace{3 - 8}_{-5} - 5 + 4 \\
 &= -5 - 5 + 4 \\
 &= \underbrace{-5 - 5}_{-10} + 4 \\
 &= -10 + 4 \\
 &= -6
 \end{aligned}$$

**Step 1: Parentheses.** There are none.**Step 2: Exponents and Roots.** Evaluate  $\sqrt{9}$  and get 3. Evaluate  $2^2$  and get 4.**Step 3: Multiplication and Division.** There are none.**Step 4: Addition and Subtraction.** Working from left to right, the subtraction  $3 - 8$  comes first. Change that subtraction to adding the opposite.Add  $3 + (-8)$  and get  $-5$ .

Working from left to right, the subtraction comes first. Change the subtraction to adding the opposite.

Add from left to right.  $-5 + (-5)$  is  $-10$ .Add  $-10 + 4$  and get  $-6$ .**Answer:** The answer is  $-6$ .

3.  $-8 + 5 - (4 \cdot 9)$

$$\begin{aligned}
 & -8 + 5 - \underbrace{(4 \cdot 9)}_{36} \\
 &= -8 + 5 - 36 \\
 &= -8 + 5 - 36 \\
 &= \underbrace{-8 + 5}_{-3} - 36 \\
 &= -3 - 36 \\
 &= -3 + -36 \\
 &= -39
 \end{aligned}$$

**Step 1: Parentheses.** Do the multiplication in the parentheses.  $4 \times 9$  is 36**Step 2: Exponents and Roots.** There are none.**Step 3: Multiplication and Division.** There are none.**Step 4: Addition and Subtraction.** Working from left to right, the addition comes first. Add  $-8 + 5$  and get  $-3$ .

Change subtraction to adding the opposite.

Add  $-3 + -36$  and get  $-39$ .**Answer:** The answer is  $-39$ .

$$\begin{aligned}
 4. \quad & 80 \div 4(-2) + \sqrt{9} \\
 & 80 \div 4(-2) + \sqrt{9} \\
 = & 80 \div 4(-2) + \underbrace{\sqrt{9}}_3 \\
 = & \underbrace{80 \div 4}_{20}(-2) + 3 \\
 = & \underbrace{20(-2)}_{-40} + 3 \\
 = & -40 + 3 \\
 = & -37
 \end{aligned}$$

Remember that the  $4(-2)$  means multiplication.

**Step 1: Parentheses.** There are no operations in parentheses.

**Step 2: Exponents and Roots.** There is a root,  $\sqrt{9}$  is 3.

**Step 3: Multiplication and Division.** Working left to right, the division comes first. Divide  $80 \div 4$  and get 20.

Multiply  $20(-2)$  and get  $-40$ .

**Step 4: Addition and Subtraction.** Add  $-40 + 3$  and get  $-37$ .

**Answer:** The answer is  $-37$ .

$$\begin{aligned}
 5. \quad & -4^2 - (2 - 14) \div (-3) \\
 & -4^2 - \underbrace{(2 - 14)}_{-12} \div (-3) \\
 = & -4^2 - \underbrace{(2 + -14)}_{-12} \div (-3) \\
 = & \underbrace{-4^2}_{-16} - (-12) \div (-3) \\
 = & -16 - \underbrace{(12) \overbrace{3}^{\cancel{-}}}_{4} \\
 = & -16 - 4 \\
 = & -16 + (-4) \\
 = & -20
 \end{aligned}$$

**Step 1: Parentheses.** There is a subtraction in the parentheses.

Change the subtraction problem  $2 - 14$  to adding the opposite.

Do the addition in the parentheses.  $2 + (-14)$  is  $-12$ .

**Step 2: Exponents and Roots.** Evaluate  $4^2$  and keep the negative sign in front. We get  $-16$ .

**Step 3: Multiplication and Division.** Divide  $(-12) \div (-3)$  which gives 4.

**Step 4: Addition and Subtraction.** Change the subtraction to adding the opposite.

Add.

**Answer:** The answer is  $-20$ .

$$\begin{aligned}
 6. \quad & 3^2 - (6 - 4 + \sqrt{64}) \times (-5) \div 2 \\
 & 3^2 - \underbrace{(6 - 4 + \sqrt{64})}_{10} \times (-5) \div 2 \\
 = & 3^2 - \underbrace{(6 - 4 + 8)}_{10} \times (-5) \div 2 \\
 = & 3^2 - \underbrace{(2 + 8)}_{10} \times (-5) \div 2 \\
 = & \underbrace{3^2}_9 - (10) \times (-5) \div 2 \\
 = & 9 - \underbrace{(10) \times (-5)}_{-50} \div 2 \\
 = & 9 - \underbrace{(-50) \div 2}_{-25} \\
 = & 9 - (-25) \\
 = & 9 + 25 \\
 = & 34
 \end{aligned}$$

**Step 1: Parentheses.** There is a subtraction, an addition, and a root in the parentheses. Do the root first,  $\sqrt{64}$  is 8.

Working left to right in the parentheses, the subtraction comes first. Do the subtraction  $6 - 4$  which is 2.

Do the addition in the parentheses.  $2 + 8$  is 10.

**Step 2: Exponents and Roots.** Evaluate  $3^2$  to get 9.

**Step 3: Multiplication and Division.** Working left to right, the multiplication comes first. Do the multiplication  $10 \times (-5)$  which is  $-50$ .

Divide  $(-50) \div 2$  is  $-25$ .

**Step 4: Addition and Subtraction.** Change the subtraction to adding the opposite.

Add.

**Answer:** The answer is 34.



$$7. \quad |-3-4|+5$$

$$= \underbrace{|-3-4|} + 5$$

$$= \underbrace{|-3+(-4)|} + 5$$

$$= \underbrace{|-7|} + 5$$

$$= 7 + 5$$

$$= 7 + 5$$

$$= 12$$

This problem contains the absolute value symbol. This operation is not specifically mentioned in the Order of Operation steps. But earlier in this section, you learned that absolute value is treated as parentheses for the purpose of order of operations. In other words, absolute value is treated as a grouping symbol just like parentheses. This means that any operations inside absolute value must be performed first.

**Absolute value is included with Parentheses in Step 1 of the Order of Operations.**

**Step 1: Parentheses.** Since absolute value is treated as parentheses, the subtraction inside the absolute value symbol must be performed first. Change the subtraction problem  $-3-4$  to adding the opposite.

Do the addition problem inside the absolute value symbol.  $-3+(-4)$  is  $-7$ .

Since there is just one number inside the absolute value symbol now, we apply the definition of absolute value. The absolute value of any non-zero number is positive. So,  $|-7|$  is 7.

**Step 2: Exponents and Roots.** There are none.

**Step 3: Multiplication and Division.** There are none.

**Step 4: Addition and Subtraction.** Do the addition problem.

**Answer:** The answer is 12.

## REVIEW: ORDER OF OPERATIONS



### PRACTICE: Evaluate.

$$1. \quad 2^3 + 30 \div 5$$

$$2. \quad 7 - 4 - (8 \cdot 3)$$

$$3. \quad 5 + |3 - 6|$$

$$4. \quad 2 - \sqrt{100} + (-9 + 4)$$

$$5. \quad 45 \div 5(-3) + \sqrt{25}$$

$$6. \quad -5^2 - (5 - 12) \cdot (-3)$$

$$7. \quad (6 - 7) + 5(32 \div 4^2)$$

$$8. \quad \sqrt{16} - (8 - 5 + 3^2) \div (-3) \times 2$$

### Answers:



<b>SECTION 1.1 SUMMARY</b>		
<b>Integers</b>		
<b>NUMBER SYSTEMS</b>	<u>Natural Numbers</u> $\{1, 2, 3, 4, 5 \dots\}$	<u>Whole Numbers</u> $\{0, 1, 2, 3, 4, 5 \dots\}$
	<u>Integers</u> $\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$ 	
<b>ABSOLUTE VALUE</b>	The absolute value of 0 is 0. The absolute value of all other real numbers is positive.	<u>Example:</u> $ 0  = 0$ <u>Examples:</u> $ 6  = 6$ $ -6  = 6$
<b>ADDITION OF INTEGERS</b>	SSS Method <ul style="list-style-type: none"> <li>• Same – signs of #s are the same</li> <li>• Sum – <b>add</b> absolute values</li> <li>• Same – answer has same sign as original #s</li> </ul> DDD Method <ul style="list-style-type: none"> <li>• Different – signs of #s are different</li> <li>• Difference – <b>subtract</b> absolute values</li> <li>• Dominant – answer has sign of bigger #</li> </ul>	<u>Example:</u> $-5 + -3$ $5 + 3 = 8$ Original #s → Negative Answer → Negative So, $-5 + -3 = -8$  <u>Example:</u> $4 + (-6)$ $6 - 4 = 2$ Bigger # → Negative Answer → Negative So, $4 + (-6) = -2$
<b>SUBTRACTION OF INTEGERS</b>	Use the “Add the Opposite” method <ul style="list-style-type: none"> <li>• Subtracting a Positive <i>becomes</i> Adding a Negative</li> <li>• Subtracting a Negative <i>becomes</i> Adding a Positive</li> </ul>	<u>Example:</u> $5 - 8 = 5 + (-8)$ <u>Example:</u> $5 - (-8) = 5 + 8$
<b>MULTIPLICATION AND DIVISION OF INTEGERS</b>	Same Signs: Positive Answer  Different Signs: Negative Answer  Use of Zero: $n \times 0 = 0$ $0 \div n = 0$ $n \div 0 = \text{undefined}$	<u>Examples:</u> $(-3) \cdot (-4) = 12$ $(-12) \div (-3) = 4$  <u>Examples:</u> $3 \times (-4) = -12$ $-12 \div 3 = -4$  <u>Examples:</u> $3 \times 0 = 0$ $0 \div 3 = 0$ $3 \div 0 = \text{undefined}$
<b>EXPONENTS</b>	$a^n \rightarrow$ the exponent ( $n$ ) specifies how many times the base ( $a$ ) is used in a repeated multiplication Note: $a^0 = 1$	<u>Examples:</u> $4^3 = (4)(4)(4) = 64$ $(-6)^2 = (-6)(-6) = 36$ $-6^2 = -(6)(6) = -36$
<b>ROOTS</b>	$\sqrt[n]{a}$ is the number that, when raised to the power $n$ , gives $a$	<u>Example:</u> $\sqrt[3]{8}$ means $\square^3 = 8$ The answer is 2 because $2^3 = 2 \times 2 \times 2 = 8$ .
<b>ORDER OF OPERATIONS</b>	Please Parentheses (and grouping symbols and absolute value) Excuse Exponents and Roots My Dear Multiplication and Division (left to right) Aunt Sally Addition and Subtraction (left to right)	<u>Example:</u> $4 + (9 - 7)^3 \times 5$ $= 4 + 2^3 \times 5$ $= 4 + 8 \times 5$ $= 4 + 40$ $= 44$

## SECTION 1.1 EXERCISES

### Integers

Circle each number system that the given number belongs to.

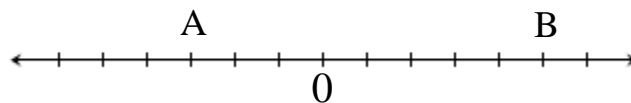
1.  $-34$       *Natural Numbers*    *Whole Numbers*    *Integers*
2.  $12$         *Natural Numbers*    *Whole Numbers*    *Integers*
3.  $0$          *Natural Numbers*    *Whole Numbers*    *Integers*

Circle each number that belongs to the specified set.

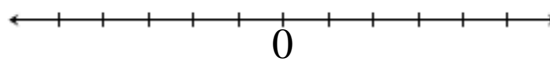
4. Circle all the integers in the list:       $-\frac{4}{5}$      $1.9$      $-8$      $3$      $-6.7$      $\frac{2}{9}$      $11$      $0$
5. Circle all the positive integers in the list:     $1.2$      $0$      $7$      $-5.8$      $9$      $\frac{1}{2}$      $-3$      $6$

Answer the following questions about points on the number line.

6. What is the value of each of the points named by the capital letters on the number line below?



7. Place the numbers 4 and  $-1$  on the number line below.



Evaluate each absolute value problem.

- |             |               |
|-------------|---------------|
| 8. $ 6 $    | 12. $ 12-8 $  |
| 9. $ -2 $   | 13. $ 2-5 $   |
| 10. $- -8 $ | 14. $- 3+6 $  |
| 11. $- 9 $  | 15. $- 16-4 $ |

Perform the indicated operation.

16.  $-27 + (-14)$

17.  $-38 + 56$

18.  $19 + (-7)$

19.  $24 + (-24)$

20.  $-3 + -17$

21.  $-16 + 11$

22.  $25 + -40$

23.  $15 - 19$

24.  $-10 - (-12)$

25.  $-20 - 8$

26.  $-17 - (-9)$

27.  $10 - (-4)$

28.  $-6 \cdot 12$

29.  $-12 \times (-4)$

30.  $-14 \cdot 0$

31.  $0(8)$

32.  $-42 \div (-7)$

33.  $90 \div (-3)$

34.  $-23 \div 0$

35.  $0 \div 56$

Evaluate each problem.

36.  $3^4$

37.  $17^0$

38.  $-12^2$

39.  $(-12)^2$

40.  $-4^3$

41.  $(-4)^3$

42.  $\sqrt{49}$

43.  $\sqrt[3]{64}$

Evaluate each expression.

44.  $5(-8)(4)$

45.  $6^1 - 3 \cdot 4$

46.  $5(\sqrt{81} - 4) \div 5 - 7$

47.  $-5^2 - 3(1 - 4) + 40$

48.  $2^2 + 3(8 \div 2^3)$

49.  $-50 \div 2 - 5^2 \cdot \sqrt{9}$

50.  $|4 - 6| + 72$